Problem-1 (40%)
Consider a geothermal supply of hot water available as saturated liquid at $P_1=1.5$ MPa and mass flow rate of 1000 kg/s. The saturated liquid is to be flashed (throttled) to some lower pressure, $P_2=100$ Kpa. The saturated vapor enters a heat exchanger and heat transfer takes place between saturated liquid and saturated vapor. It is assumed that there is no entropy change through the process of ground water in heat exchanger. Vapor is expanded through a reversible adiabatic turbine to the exhaust pressure, $P_3=10$ KPa and the quality at exit state of ideal turbine is 0.9285.

a) Label the states at T-s diagram provided below.
b) Calculate the isentropic turbine work output rate.
c) Assume that the exit state of the actual turbine is saturated vapor. Calculate the efficiency of turbine.
d) Calculate thermal efficiency.
a) \( P_1 = 1500 \text{Kpa} \)

\( h_1 = h_2 = h_f = 844.87 \text{kJ/kg} \),

@ \( P_2 = 100 \text{Kpa} \)

\( h_g = 2675.46 \text{kJ/kg} = h_y \), \( h_f = 417.44 \text{kJ/g} \), \( h_{fg} = 2258.02 \text{kJ/kg} \),

\( s_f = 1.3025 \text{kJ/L/kg} \), \( s_g = 7.3593 \text{kJ/kg} \)

\[ x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{844.87 - 417.44}{2258.02} = 0.189 \]

\( s_i = s_4 = 2.3150 \text{kJ/kg} \)

\[ x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{2.3150 - 1.3025}{6.0568} = 0.167 \]

\[ h_4 = h_f + (x_4) h_{fg} = 417.44 + (0.167)2258.02 = 794.53 \text{kJ/kg} \]

\[ \text{let}\ m\ \text{kg/s is flow rate of liquid entering throttle.} \]

\[ \text{So, } (1000 - m) \text{kg/s is flow rate of liquid entering heat exchanger.} \]

\[ x_2 = 0.189 = \frac{m_{vap}}{m}, \quad m_{vap} = (0.189)m \]

@ \( P_2 = 10 \text{Kpa} \)

\( h_g = 2584.63 \text{kJ/kg} \), \( h_f = 191.81 \text{kJ/g} \), \( h_{fg} = 2392.82 \text{kJ/kg} \)

\( s_f = 0.6492 \text{kJ/kg} \), \( s_g = 8.1501 \text{kJ/kg} \)

\[ x_y = \frac{s_y - s_f}{s_{fg}} = \frac{s_y - 0.6492}{7.5010} = 0.9285 \]

\[ h_y = h_f + (x_y) h_{fg} = 191.81 + (0.9285)2392.82 = 2413.54 \text{kJ/kg} \]

\( s_y = 7.6138, \quad s_{zv} = s_y \); \text{Superheated table } @\ P_2 = 100 \text{Kpa} \), \( T = 150^\circ \text{C} \)

\( h_{zv} = 2776.38 \text{kJ/kg} \)

\text{System: Heat exchanger}

\text{Heat balance equation between hot liquid and saturated vapor:}

\[ (1000 - m)(h_f - h_y) = (0.189)m(h_{zv} - h_y) \]

\[ (1000 - m)(844.87 - 794.53) = (0.189)m(2776.38 - 2675.46) \]

\[ m = 725 \text{kg/s} \]

b) \text{System: Turbine}

\[ W_T = m_{vap} (h_{zv} - h_y) = (0.189)m(2776.38 - 2413.54) \]

\[ W_T = m_{vap} (h_{zv} - h_y) = (0.189)725(2776.38 - 2413.54) = 49.7 \text{MW} \]

c) & d) \text{Turbine efficiency & thermal efficiency of the plant:}

\[ \eta_T = \frac{W_{act}}{W_{ide}} = \frac{(0.189)725(h_{zv} - h_y)}{49718} = \frac{26274}{49718} = 52.8\% \]

\[ \eta_{TH} = \frac{\text{Benefit}}{\text{Cost}} = \frac{W_{act}}{Q_{HT}} = \frac{(0.189)725(h_{zv} - h_y)}{1000(844.87)} = \frac{26274}{844870} = 3.1\% \quad (\text{thermal efficiency is very low. However, energy cost is indeed almost zero. It is free from mother nature. So, 3.1 \% thermal efficiency is very meaningful.}) \]
Problem-2 (15%)  
Consider the constant transfer of energy from a warm room at 22.9°C inside of house to the colder ambient at -7.04°C through a single-pane window as shown in Fig below. The glass pane has a thickness of 5mm with a conductivity of 1.4 W/m K and a total surface area of 1.0 m². The outside wind is blowing so that convective heat transfer coefficient is 100 W/m²K. With the outer glass surface temperature of 15°C, find the emissivity of the glass pane?

Solution:

\[
\dot{Q}_{\text{Cond}} = -kA \frac{dT}{dx} = 1.4 \times 1.0 \times \frac{22.9 - 15}{0.005} = -2212W \quad (\text{\(\bullet\)} \text{ sign means energy lost.})
\]

\[
\dot{Q}_{\text{Con}} = Ah\Delta T = 1.0 \times 100 \times (15 - (-7.04)) = 2204W
\]

\[
\dot{Q}_{\text{Rad}} = \varepsilon \sigma A T_s^4 = \varepsilon \times 5.67 \times 10^{-8} \times (-8) \times 1.0 \times (273.15 + 15)^4 = \left| \dot{Q}_{\text{Cond}} \right| - \dot{Q}_{\text{Con}}
\]

\[
\dot{Q}_{\text{Rad}} = \varepsilon \sigma A T_s^4 = \varepsilon \times 390.9 = 2212 - 2204 = 8W
\]

\[
\varepsilon = 0.02
\]
Problem-3 (15%)  
A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the maximum temperature is 1600 K. The minimum pressure in the cycle is 100 KPa, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?

Solution:

Brayton cycle so this means:
- Minimum T: \( T_1 = 300 \text{ K} \)
- Maximum T: \( T_3 = 1600 \text{ K} \)
- Pressure ratio: \( P_2/P_1 = 14 \)

Solve using constant \( C_{p0} \)

Compression in compressor: \( s_2 = s_1 \Rightarrow \) Implemented in Eq.8.32

\[
T_2 = T_1 (P_2/P_1)^{\frac{k-1}{k}} = 300(14)^{0.286} = 638.1 \text{ K}
\]

\[
w_C = h_2 - h_1 = C_{p0}(T_2 - T_1) = 1.004 (638.1 - 300) = 339.5 \text{ kJ/kg}
\]

Expansion in turbine: \( s_4 = s_3 \Rightarrow \) Implemented in Eq.8.32

\[
T_4 = T_3 (P_4/P_3)^{\frac{k-1}{k}} = 1600 (1/14)^{0.286} = 752.2 \text{ K}
\]

\[
w_T = h_3 - h_4 = C_{p0}(T_3 - T_4) = 1.004 (1600 - 752.2) = 851.2 \text{ kJ/kg}
\]

\[
w_{NET} = 851.2 - 339.5 = 511.7 \text{ kJ/kg}
\]

Do the overall net and cycle efficiency

\[
\dot{m} = \frac{\dot{W}_{NET}}{w_{NET}} = 100000/511.7 = 195.4 \text{ kg/s}
\]

\[
\dot{W}_T = \dot{m}w_T = 195.4 \times 851.2 = 166.32 \text{ MW}
\]

\[
w_C/w_T = 339.5/851.2 = 0.399
\]

Energy input is from the combustor

\[
q_H = C_{p0}(T_3 - T_2) = 1.004 (1600 - 638.1) = 965.7 \text{ kJ/kg}
\]

\[
\eta_{TH} = \frac{w_{NET}}{q_H} = 511.7/965.7 = 0.530
\]
Problem-4 (15%)  
A gasoline engine has a volumetric compression ratio of 10 and before compression has air at 290 K, 85 KPa in the cylinder. The combustion peak pressure is 6000 KPa. Assume cold air properties. What is the highest temperature in the cycle? Find the temperature at the beginning of the exhaust (heat rejection) and the overall cycle efficiency.

Solution:
Compression. Isentropic so we use Eqs.8.33-8.34
\[ P_2 = P_1 \left(\frac{v_1}{v_2}\right)^k = 85 \times (10)^{1.4} = 2135.1 \text{ kPa} \]
\[ T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{k-1} = 290 \times (10)^{0.4} = 728.45 \text{ K} \]
Combustion. Constant volume
\[ T_3 = T_2 \left(\frac{P_3}{P_2}\right) = 728.45 \times 6000/2135.1 = 2047 \text{ K} \]
Exhaust. Isentropic expansion so from Eq.8.33
\[ T_4 = T_3 / \left(\frac{v_1}{v_2}\right)^{k-1} = T_3 / 10^{0.4} = 2047 / 2.5119 = 814.9 \text{ K} \]
Overall cycle efficiency is from Eq.11.18, \( r_v = v_1/v_2 \)
\[ \eta = 1 - r_v^{1-k} = 1 - 10^{-0.4} = 0.602 \]
Comment: No actual gasoline engine has an efficiency that high, maybe 35%.
Problem 5 (15%)
A refrigerator in a meat warehouse must keep a low temperature of -15°C and the outside temperature is 20°C. It uses R-12 as the refrigerant, which must remove 5 kW from the cold space. Find the flow rate of the R-12 needed assuming a standard vapor compression refrigeration cycle with a condenser at 20°C.

Solution:
Basic refrigeration cycle: \( T_1 = T_4 = -15^\circ C, \quad T_3 = 20^\circ C \)

Table B.3: \( h_4 = h_3 = 54.87 \text{ kJ/kg}; \quad h_1 = h_2 = 180.97 \text{ kJ/kg} \)

\[
\dot{Q}_L = \dot{m}_{R-12} \times q_L = \dot{m}_{R-12} (h_1 - h_4)
\]

\[
q_L = 180.97 - 54.87 = 126.1 \text{ kJ/kg}
\]

\[
\dot{m}_{R-12} = \frac{5.0}{126.1} = 0.03965 \text{ kg/s}
\]

Ideal refrigeration cycle
\( T_{\text{cond}} = 20^\circ C \)
\( T_{\text{evap}} = -15^\circ C = T_1 \)

Properties from Table B.3