P = $100 (P/A, 12%, 5) + $50 (P/G, 12%, 5) = $100 (3.605) + $50 (6.397) = $680.35

F = $680.35 (F/P, 12%, 5)
   = $680.35 (1.762)
   = $1,198.78

Alternate Solution

F = [$100 + $50 (A/G, 12%, 5)] (F/A, 12%, 5)
   = [$100 + $50 (1.775)] (6.353)
   = 1,199.13
9-3

\[ F = 5 \times (P/G, 10\%, 6) \times (F/P, 10\%, 12) + 30 \times (F/A, 10\%, 6) \]
\[ = 5 \times (6.684) \times (3.138) + 30 \times (7.716) \]
\[ = 383.42 \]

9-9

\[ F = 30,000 \times (F/P, 10\%, 15) + 600 \times (F/A, 10\%, 15) \]
\[ = 30,000 \times (4.177) + 600 \times (31.772) \]
\[ = 144,373 \]
F = $3,200 (F/A, 7%, 30) + $60 (P/G, 7%, 30) (F/P, 7%, 30)
= $3,200 (94.461) + $60 (120.972) (7.612)
= $357,526
\[ x = \text{years to continue working} \]
\[ \text{age to retire} = 55 + x \]

Amount at Retirement = PW of needed retirement funds

\[
\begin{align*}
48,500 \left( \frac{F}{P}, 12\%, x \right) + 5,000 \left( \frac{F}{A}, 12\%, x \right) \\
= 20,000 \left( \frac{P}{A}, 12\%, 21-x \right)
\end{align*}
\]

**Try \( x = 10 \)**

\[
\begin{align*}
48,500 \left( 3.106 \right) + 5,000 \left( 17.549 \right) &= 238,386 \\
20,000 \left( 5.938 \right) &= 118,760 \text{ so } x \text{ can be } < 10
\end{align*}
\]

**Try \( x = 5 \)**

\[
\begin{align*}
48,500 \left( 1.762 \right) + 5,000 \left( 6.353 \right) &= 117,222 \\
20,000 \left( 6.974 \right) &= 139,480 \text{ so } x > 5
\end{align*}
\]

**Try \( x = 6 \)**

\[
\begin{align*}
48,500 \left( 1.974 \right) + 5,000 \left( 8.115 \right) &= 136,314 \\
20,000 \left( 6.811 \right) &= 136,220
\end{align*}
\]

Therefore, \( x = 6 \). The youngest age to retire is \( 55 + 6 = 61 \).
FW (Costs) = $150,000 (F/P, 10%, 10) + $1,500 (F/A, 10%, 10) + $500 (P/G, 10%, 7) (F/P, 10%, 7) – (0.05) ($150,000)
= $150,000 (2.594) + $1,500 (15.937) + $500 (12.763) (1.949) – $7,500
= $417,940

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th></th>
<th>B</th>
<th></th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$600</td>
<td></td>
<td>$500</td>
<td></td>
<td>$200</td>
</tr>
<tr>
<td>Uniform Annual Benefit</td>
<td>$158.3</td>
<td></td>
<td>$138.7</td>
<td></td>
<td>$58.3</td>
</tr>
</tbody>
</table>

B/COFA = $158.3/[$600 (A/P, 10%, 5)] = 1.00
B/COFB = $138.7/[$500 (A/P, 10%, 5)] = 1.05
B/COFC = $58.3/[$200 (A/P, 10%, 5)] = 1.11
All alternatives have a B/C ratio ≥ 1.00. Proceed with incremental analysis.

<table>
<thead>
<tr>
<th></th>
<th>B – C</th>
<th>A – B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$300</td>
<td>$100</td>
</tr>
<tr>
<td>Uniform Annual Benefit</td>
<td>$8034</td>
<td>$19.6</td>
</tr>
</tbody>
</table>

B/COFB–C = $80.4/[$300 (A/P, 10%, 5)] = 1.02
Desirable increment. Reject C.

B/COFA–B = $19.6/[$100 (A/P, 10%, 5)] = 0.74
Undesirable increment. Reject A.

Conclusion: Select B.
Costs = Benefits at end of year 8
Therefore, payback period = 8 years.

9-43

**Lease**: A = $5,000/yr

**Purchase**:

\[ S = \$500 \]

- **$7,000**
- **$3,500**

(a) Payback Period
- Cost = $7,000
- Benefit = $1,500/yr + $500 at any time
- Payback = \( \frac{(7,000 - 500)}{1,500} = 4.3 \text{ years} \)

(b) Benefit-Cost Ratio
- \( B/C = \frac{EUAB}{EUAC} \)
- \( = \frac{[1,500 + 500 (A/F, 10\%, 6)]}{[7,000 (A/P, 10\%, 6)]} \)
- \( = \frac{[1,500 + 500 (0.1296)]}{[7,000 (0.2296)]} \)
- \( = 0.97 \)
(a) Solve by Future Worth analysis. In future worth analysis there must be a common future time for all calculations. In this case, 12 years hence is a practical future time.

**Alternative A**

\[ NF_{WA} = \$28.8 \left( F/A, 12\%, 12 \right) - \$50 \left( A/P, 12\%, 2 \right) \left( P/A, 12\%, 12 \right) \]
\[ = \$28.8 \left( 24.133 \right) - \$50 \left( 0.5917 \right) \left( 24.133 \right) \]
\[ = \$18.94 \]

**Alternative B**

\[ NF_{WB} = \$39.6 \left( F/A, 12\%, 12 \right) - \$150 \left( F/P, 12\%, 6 \right) - \$150 \left( F/P, 12\%, 12 \right) \]
\[ = \$39.6 \left( 24.133 \right) - \$150 \left( 1.974 + 3.896 \right) \]
\[ = +$75.17 \]

**Alternative C**

\[ NF_{WC} = \$39.6 \left( F/A, 12\%, 12 \right) - \$110 \left( F/P, 12\%, 4 \right) - \$110 \left( F/P, 12\%, 8 \right) - \$
\[ = \$39.6 \left( 24.133 \right) - \$110 \left( 1.574 + 2.476 + 3.896 \right) \]
\[ = +$81.61 \]

Choose Alternative C because it maximizes Future Worth.
(a) Payback_A = 4 years
Payback_B = 2.6 years
Payback_C = 2 years

To minimize payback, choose C.

(b) B/C Ratios:
B/C_A = ($100 (P/A, 10%, 6) + $100 (P/F, 10%, 1))/$500
   = 1.05
B/C_B = ($125 (P/A, 10%, 5) + $75 (P/F, 10%, 1))/$400
   = 1.36
B/C_C = ($100 (P/A, 10%, 4) + $100 (P/F, 10%, 1))/$300
   = 1.36

**Incremental Analysis**

<table>
<thead>
<tr>
<th>Year</th>
<th>B - C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$100</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>+$25</td>
</tr>
<tr>
<td>3</td>
<td>+$25</td>
</tr>
<tr>
<td>4</td>
<td>+$25</td>
</tr>
<tr>
<td>5</td>
<td>+$125</td>
</tr>
</tbody>
</table>

\[ \Delta B/\Delta C_{B-C} = \frac{($25 (P/A, 10\%, 3)(P/F, 10\%, 1) + $125 (P/F, 10\%, 5))}{$100} \]
\[ = 1.34 \]

This is a desirable increment. **Reject C.**

A - B Increment

<table>
<thead>
<tr>
<th>Year</th>
<th>A - B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$100</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>-$25</td>
</tr>
<tr>
<td>3</td>
<td>-$25</td>
</tr>
<tr>
<td>4</td>
<td>-$25</td>
</tr>
<tr>
<td>5</td>
<td>+$100</td>
</tr>
</tbody>
</table>

By inspection we see that \( \Delta B/\Delta C < 1 \)
\[ \Delta B/\Delta C_{A-B} = \frac{($100 (P/F, 10\%, 6))}{($100 + $25 (P/A, 10\%, 4)(P/F, 10\%, 1))} \]
\[ = 0.33 \]

**Reject A.**

**Conclusion:** Select B.