Time Value of Money means simply that "money has value over time." Money has value, of course, because of what it can purchase. However, the time value of money means that ownership of money is valuable, and it is valuable because of the interest dollars that can be earned/gained due to its ownership. Understanding interest and its impact is important in many life circumstances. Examples could include some of the following:

- Selecting the best loans for homes, boats, jewelry, automobiles, etc.
- Many aspects involved with businesses ownership (payroll, taxes, etc.)
- Using the best strategies for paying off personal loans, credit cards, debt
- Making investments for life goals (purchases, retirement, college, weddings, etc.)
- Etc.

$2,000 + $2,000 \times 0.10 \times 3 = \$2,600$

$Q = \$200 \times (P/F, 10\%, 4) = \$200 \times 0.683 = \$136.60$
3-8

\[ P = \$750, \quad n = 3 \text{ years}, \quad i = 8\%, \quad F = ? \]
\[
F = P \left(1 + i\right)^n = \$750 \left(1.08\right)^3 = \$750 \left(1.260\right)
= \$945
\]

Using interest tables:
\[
F = \$750 \left(F/P, 8\%, 3\right) = \$750 \left(1.360\right)
= \$945
\]

3-10

Use \( F = P \left(F/P, i, n\right) = P \left(1 + i\right)^n = 2000 \left(1 + 0.06\right)^n \).

(a) \( n = 5 \), \( F = \$2,676 \)
(b) \( n = 10 \), \( F = \$3,582 \)
(c) \( n = 20 \), \( F = \$6,414 \)
(d) \( n = 50 \), \( F = \$36,840 \)
(e) \( n = 100 \), \( F = \$678,604 \)

3-18

\[ P = \$1,400 \left(P/A, 10\%, 5\right) - \$80 \left(P/G, 10\%, 5\right) \]
\[
= \$1,400 \left(3.791\right) - \$80 \left(6.862\right)
= \$4,758.44
\]

Using single payment factors:
\[
P = \$1400 \left(P/F, 10\%, 1\right) + \$1,320 \left(P/F, 10\%, 2\right) + \$1,240 \left(P/F, 10\%, 3\right) + \$1,160 \left(P/F, 10\%, 4\right) + \$1,080 \left(P/F, 10\%, 5\right)
= \$1,272.74 + \$1,090.85 + \$931.61 + \$792.28 + \$670.57
= \$4,758.05
3-20

\[ P = \$1, \ n = \text{unknown number of semiannual periods}, \ i = 2\%, \ F = 2 \]

\[ F = P (1 + i)^n \]
\[ 2 = 1 (1.02)^n \]
\[ n = \log (2) / \log (1.02) \]
\[ = 35 \]

Therefore, the money will double in 17.5 years.

3-21

**Calculator Solution**

1\% per month \[ F = \$1,000 \ (1 + 0.01)^{12} \] \[ = \$1,126.83 \]

12\% per year \[ F = \$1,000 \ (1 + 0.12)^1 \] \[ = \$1,120.00 \]

Savings in interest = $6.83

**Compound interest table solution**

1\% per month \[ F = \$1,000 \ (1.127) \] \[ = \$1,127.00 \]

12\% per year \[ F = \$1,000 \ (1.120) \] \[ = \$1,120.00 \]

Savings in interest = $7.00

3-22

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-$2,000</td>
</tr>
<tr>
<td>1</td>
<td>-$4,000</td>
</tr>
<tr>
<td>2</td>
<td>-$3,625</td>
</tr>
<tr>
<td>3</td>
<td>-$3,250</td>
</tr>
<tr>
<td>4</td>
<td>-$2,875</td>
</tr>
</tbody>
</table>
Given two cash flows that are equivalent if the interest rate is $i$. Which one is more valuable if the interest rate is $2i$?

For rate $i$: \[ P_1 = F_1 \left( \frac{P}{F}, i, 2 \right) = F_2 \left( \frac{P}{F}, i, 3 \right) = P_2 \]
\[ F_2 = F_1 \left( \frac{P}{F}, i, 2 \right) = F_1 \left( \frac{(1+i)^2}{(1+i)^3} \right) = F_1 (1+i) \]

For rate $2i$: \[ P'_1 = F_1 \left( \frac{P}{F}, i, 2 \right) \text{ and } P'_2 = F_2 \left( \frac{P}{F}, i, 3 \right) = F_1 (1+i)(1+2i)^{-3} \]
\[ \frac{P'_2}{P'_1} = \frac{F_1 (1+i)(1+2i)^{-3}}{F_1 (1+2i)^{-2}} = \frac{(1+i)(1+2i)^2}{(1+2i)^3} = \frac{1+i}{1+2i} < 1 \]
so \[ P'_2 < P'_1 \]

Thus, the cash flow in diagram i is more valuable than the cash flow in diagram ii.

Example: Let $F_1 = 1000$ and $i = 10\%$ then $F_2 = (1000)(1 + 0.1)^1 = 1100$. At $i = 2i = 20\%$ we have \[ P'_1 = 1000 \left( 1 + 0.2 \right)^2 = 694.4 \]
\[ P'_2 = 1100 \left( 1 + 0.2 \right)^{-3} = 636.6 \]
$Q_6$

$Q_{10}$

$i = 5\%$

$P = $60

Either:

$Q_{10} = Q_6 \ (F/P, \ 5\%, \ 4) \ (1)$

$Q_{10} = P \ (F/P, \ 5\%, \ 10) \ (2)$

Since $P$ is between and $Q_6$ is not, solve Equation (2),

$Q_{10} = $60 $\times (1.629)$

$= $97.74

3-30

The garbage company sends out bills only six times a year. Each time they collect one month's bills one month early.

100,000 customers $\times$ $6.00 \times 1\% \ per \ month \times 6 \ times/yr = $36,000