Problem-1 (3%)

The breaking strengths of a random sample of 26 molded plastic housings were measured, and a sample mean of \( \bar{x} = 479.42 \) and a sample standard deviation of \( s = 12.55 \) were obtained. A confidence interval (472.56, 486.28) for the average strength of molded plastic housings was constructed from these results. What is the confidence level of this confidence interval?

The interval (472.56, 486.28) is \((479.42 - 6.86, 479.42 + 6.86)\), and \(6.86 = \frac{2.787 \times 12.55}{\sqrt{26}}\).
Since \(2.787 = t_{0.005,25}\) it follows that the confidence level is \(1 - (2 \times 0.005) = 0.99\).

Problem-2 (3%)

Composites are materials that are made by embedding a fiber, such as glass or carbon, inside a matrix, such as a metal or a ceramic. Composites are used in civil engineering structures, and their degradation when subjected to weather conditions is an important issue. In an experiment to investigate the effect of moisture on a certain kind of composite, the weight gains of a collection of 18 samples of composite subjected to water diffusion were obtained. The sample mean was \(\bar{x} = 0.337\%\), with a sample standard deviation of \(s = 0.025\%\).

(a) Is it safe to conclude from the results of this experiment that the average weight gain for composites of this kind is smaller than 0.36%?

(b) Construct a 99% confidence interval that provides an upper bound for the average weight gain for composites of this kind.

(a) Consider the hypotheses \(H_0 : \mu \geq 0.36\) versus \(H_A : \mu < 0.36\).

The test statistic is
\[
t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{18}(0.337 - 0.36)}{0.025} = -3.903.
\]

The p-value is
\[
P(X \leq -3.903) = 0.0006
\]
where the random variable \(X\) has a \(t\) distribution with \(18 - 1 = 17\) degrees of freedom.

There is sufficient evidence to conclude that the average weight gain for composites of this kind is smaller than 0.36%.

(b) Using the critical point \(t_{0.01,17} = 2.567\) the confidence interval is
\[
(-\infty, 0.337 + \frac{2.567 \times 0.025}{\sqrt{18}}) = (-\infty, 0.352).
\]
Problem-3 (1.5%) 
Probability density function vs. failure times of battery plotted as below. Compute Expected failure time of battery?

Problem-4 (3%) 
The thicknesses of glass sheets produced by a certain process are normally distributed with a mean of $\mu = 3.00$ mm and a standard deviation of $\sigma = 0.12$ mm.

(a) What is the probability that a glass sheet is thicker than 3.2 mm?
(b) What is the probability that a glass sheet is thinner than 2.7 mm?
(c) What is the value of $c$ for which there is a 99% probability that a glass sheet has a thickness within the interval $[3.00 - c, 3.00 + c]$?

(a) $P( X \geq 3.2 ) = 0.0478$.  
(b) $P( X \leq 2.7 ) = 0.0062$. 
(c) Solving $P(3.00 - c \leq X \leq 3.00 + c) = 0.99$ gives $c = 0.12 \times z_{0.005} = 0.12 \times 2.5758 = 0.3091$. 

Problem-5 (3%)
The amount of sugar contained in 1-kg packets is actually normally distributed with a mean of \( \mu = 1.03 \text{ kg} \) and a standard deviation of \( \sigma = 0.014 \text{ kg} \).

(a) What proportion of sugar packets are underweight?

(b) If an alternative package-filling machine is used for which the weights of the packets are normally distributed with a mean of \( \mu = 1.05 \text{ kg} \) and a standard deviation of \( \sigma = 0.016 \text{ kg} \), does this result in an increase or a decrease in the proportion of underweight packets?

(c) In each case, what is the expected value of the excess package weight above the advertised level of 1 kg?

(a) \( P( X \leq 1) = 0.0161 \).

(b) \( P( X \leq 1) = 0.0009 \).

There is a decrease in the proportion of underweight packets.

(c) The expected excess weight is \( \mu - 1 \) which is 0.03 and 0.05.

**Problem-6 (2.5%)**

In a certain circuit design, it is important that standard deviation of the current be less than 15 mA. In a test of twelve parts, the sample standard deviation is found to be 10 mA. Can we be 99% confident that the standard deviation will not exceed 15 mA?

Chi squared distribution.
S = 10, n = 12, v = 11. \( \alpha = 1-0.99 = 0.01 \), \( \alpha/2 = 0.005 \), \( 1-\alpha/2 = 0.995 \).
\( \chi^2_{\alpha/2} = 26.757 \), \( \chi^2_{1-\alpha/2} = 2.6032 \).

\[ \frac{(n-1)S^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}} \]

\[ \frac{12-1)10^2}{26.757} \leq \sigma^2 \leq \frac{(12-1)10^2}{2.6032} \]

41.11 \( \leq \sigma^2 \leq 422.5 \) or \( 6.41 \leq \sigma \leq 20.55 \)
We cannot be 99% confident that the standard deviation is less 15 mA.

**Problem-7 (4%)**

A simple spring is used to measure force. The spring is considered to be linear, so that \( F = kx \), where F is the force in Newtons, k is the spring constant in Newton per centimeter, and x is displacement in cm. If \( x = 12.5 \pm 1.25 \text{ cm} \) and \( k = 700 \pm 18 \text{ N/cm} \), calculate the max. possible error and the uncertainty of the measured force in absolute (dimensional) and relative (%) terms.

\( F = kx \)

\[ (w_F)_{\text{max}} = \left| w_k \frac{\partial F}{\partial k} + w_x \frac{\partial F}{\partial x} \right| \]

\[ \frac{\partial F}{\partial x} = k = 700 \text{ N/ cm} \]

\[ \frac{\partial F}{\partial k} = x = 12.5 \text{ cm} \]

\( (w_F)_{\text{max}} = (18 \times 12.5 + 1.25 \times 700) \)

\( (w_F)_{\text{max}} = 1,100 \text{ N} \)

\[ w_F = \sqrt{(w_k \frac{\partial F}{\partial k})^2 + (w_x \frac{\partial F}{\partial x})^2} \]

\[ = \sqrt{(18 \times 12.5)^2 + (1.25 \times 700)^2} \]

\[ = 903 \text{ N} \]

\( F = 700 \times 12.5 = 8,750 \text{ N (absolute value)} \)

\[ \frac{w_F}{F} \times 100 = \frac{903}{8750} \times 100 = 10.3\% \text{ (relative value)} \]