Problem-1
Pressurized air are being used in a manufacturing company in its daily operation. There are three compressors in service; two-screw type compressors (S1, S2) and one-reciprocal type compressor (R).
Each compressor, at any given time, is either off (0), or on (1).
Assume that event A is denoted to reciprocal compressor (R) that is always off. Event B is denoted that at least one of the screw type of compressor is always on. Assume that all outcomes have equal probability to occur. Compute P(A ∩ B)

Solution:

\[
P(A \cap B) = 3 \times \frac{12.5}{10} = 37.5\%
\]

Ans:

37.5%

Problem-2
A company manufacture metal cylinders that the diameter of metal cylinder has a probability density function of
\( f(x) = 1.5 - 6(x - 50.00)^2 \). What is the probability that the diameter of any randomly selected cylinder is \( x \leq 50.00 \) mm?

Solution:

\[
P(x \leq 50) = 50\%
\]

Ans:

50%

Problem-3
Hospital Emergency room accepts an average of about 21 bone fracture patients a week that required resources available with this demand accordingly. What is the probability that on any given day more than one fracture patients arrive at the emergency room?

Solution:

\[
a = \frac{21}{4} = 3
\]

\[
P(x > 1) = 1 - P(x = 0) - P(x = 1) = 80\%
\]

\[
P(x = 0) \approx 5\%
\]

\[
P(x = 1) = 15\%
\]

Ans:

80%

Problem-4
An Air Force intercept squadron consists of 16 planes that should always be ready for immediate launch. However a plane’s engines are troublesome, and there is a probability of 25% that the engines of a particular plane will not start at a given attempt.
What is the probability that all planes would successfully launched at any given attempt?

Solution:

\[
P(x = 0) = \left( \frac{16}{16} \right) \left( 0.75 \right)^0 \left( 0.25 \right)^{16} = 1\%
\]

Ans:

1\%
Problem-5
A paper company sells paper that is supposed to have a weight of 75.0 g/m². In a quality inspection, the weights of the 29 random samples of paper are measured. The sample mean of these weights is \( \bar{x} = 74.23 \) g/m² with a sample standard deviation of \( s = 2.05 \) g/m². Can we be confident with 90% confidence level that paper has an average weight of 75 g/m²? Justify your answer!

Solution: 
Yes, if 90\% conf. interval includes 75 g/m²?

\[ a = 10\% \]

\[ t_{0.1/2} = 1.701 \]

\[ \frac{2.05}{\sqrt{29}} \]

\[ 74.88 < 75 \]

[No]

Ans:
No; upper bound < 75 g/m².

Problem-6
When it is operating properly, a chemical plant has a daily production rate that is normally distributed with a mean of 885 tons/day and a standard deviation of 42 tons/day. During an analysis of the plant, the output is measured with random sampling on 60 consecutive days, and the mean output is found to be \( \bar{x} = 875 \) tons/day. The manager claims that at least 95% probability that the plant is operating properly. Is he right? Justify your answer.

Solution:
Yes, if 95\% conf. interval includes 885 tons/day

\[ \frac{z}{p} = 1.96 / \frac{42}{\sqrt{60}} = 10.63 \]

\[ \mu = 875 + 10.63 > 885 \text{ tons/day} \]

Yes

Ans:
Yes; bound > 885 tons/day.

Problem-7
Sample of Aluminum channels was tested for stiffness. The following frequency distribution was obtained. The distribution is assumed to be normal. What is the approximate probability that stiffness would be more than 2480 for any given channel section?

<table>
<thead>
<tr>
<th>Stiffness</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2400</td>
<td>20</td>
</tr>
<tr>
<td>2440</td>
<td>35</td>
</tr>
<tr>
<td>2480</td>
<td>40</td>
</tr>
<tr>
<td>2360</td>
<td>35</td>
</tr>
<tr>
<td>2320</td>
<td>20</td>
</tr>
</tbody>
</table>

Solution:

\[ \bar{x} = 2400 \]

\[ S = 49.7 \]

\[ Z = \frac{2480 - 2400}{49.7} = 1.61 \]

\[ P(Z > 1.61) = 0.5 - 0.4463 = 5.37\% \]

Ans:
5.37%.

Problem-8
To avoid any high-speed imbalance in a rotating shaft, it needs to have a variance of its diameter below 0.0004 mm². The shaft diameter has been measured at 16 equally distributed locations, with a standard deviation of 0.018 mm. Is the shaft acceptable to the customer with 95% conf. level? Justify your answer.

Solution:
Yes, if 95\% conf. interval of lower bound for variance \( \sigma^2 \) greater than 0.0004 mm².

\[ \chi^2_{0.025} = 27.488 \]

\[ 27.488 \leq \frac{0.0004}{s^2} \]

\[ \Rightarrow s^2 = \frac{0.0004}{27.488} \]

\[ \Rightarrow \sigma^2 = \frac{0.0004}{27.488} \]

[No]

Ans:
No; bound > 0.0004.
Problem 9
In a cheese factory, 5.0 kg blocks of cheese are cut manually. For a large number of blocks, the standard deviation of the cutting process is measured and found to be 0.11 kg. The measurement was done with a scale with an accuracy of 1.0% of the full scale of 15 kg. Calculate the total uncertainty of the weight of the blocks of cheese at a 95% confidence level.

Solution:
\[ s = 0.11 \text{ kg} \]
\[ \text{for large sample } t = 2 \]
\[ s_t = (0.11)(2) = 0.22 \text{ kg} \]
\[ \text{Ans: } 0.27 \text{ kg} \]

Problem 10
A single strain gage has a nominal resistance of 120 ohm. For a quarter bridge with 120 ohm fixed resistors, the voltage output is 1.658 mV with a strain of 1000-\text{\mu} strain for a supply voltage of 3V. Compute gage factor.

Solution:
\[ S = \frac{1.658 \times (120+120)^2}{3 \times 1000 \times 10^{-6} \times 120 \times 120} \]
\[ S \approx 2.2 \]

Ans: 2.2

Problem 11
The data set of the diameters of the metal cylinders manufactured an automatic machine has \( n = 30 \), mean of \( x = 49.92 \) and, std. \( S = 0.16 \). Expected diameter of metal cylinder is \( \mu = 50 \)
Is there any doubt that machine is miscalibrated? Justify your answer!

Solution:
Yes, if \( \alpha = 1 \) (at least), or 99% conf. interval would not include 50? \( t_{.005} = 2.756 \)
\[ \nu = 29 \]
\[ \text{Ans: } \text{No; bound } > 50 \]

Problem 12
A company ordered well manometers, which required accuracy is 0.067% of the max. reading. The max. uncertainty labeled on manometer (+/-) 20.1 ps. The reading scale uncertainty is 0.10 mm. The max. Allowable uncertainty in the gauge fluid density is (+/-) 0.05%. Compute the density of fluid gauge and identify the fluid in the gauge.

Solution:
\[ \text{max. reading} = \frac{(20.1)(140)}{0.067} \]
\[ (20.1)^2 = \left( \frac{(gh)(w_p)}{2} \right)^2 + \left( \frac{(gh)(w_h)}{2} \right)^2 \]
\[ 404.01 = \left[ \frac{0.05 \times 10^{-2}}{g} \right]^2 + \left[ 9.62 \times 10^{-7} \right]^2 \]
\[ 404.01 = 15^2 + 9.62 \times 10^{-7} \]
\[ \rho = 13641 \text{ kg/m}^3, H_2 \]

Ans: \[ \rho = 13641 \text{ kg/m}^3 \]
Problem 13
In using a temperature probe, following uncertainties were determined:

<table>
<thead>
<tr>
<th>Type of Error</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hysteresis</td>
<td>(+/-) 0.15 C</td>
</tr>
<tr>
<td>Linearity error</td>
<td>(+/-)0.25% of the reading</td>
</tr>
<tr>
<td>Repeatability</td>
<td>(+/-) 0.1 C</td>
</tr>
<tr>
<td>Resolution error</td>
<td>(+/-) 0.023 C</td>
</tr>
<tr>
<td>Zero offset</td>
<td>(+/-) 0.1 C</td>
</tr>
</tbody>
</table>

Determine type of these errors (random or systematic) and total uncertainty due to these effects for a temperature reading of 120 C.

Solution:
\[
\begin{align*}
\Delta T &= \sqrt{0.15^2 + 0.025^2 + 0.01^2} = 0.35 C \\
\Delta P &= \sqrt{0.1^2 + 0.0025^2} = 0.103 C \\
\Delta \omega &= 0.36 C , \Delta \phi = 0.2 ^\circ C
\end{align*}
\]

Ans:
0.36 C

Problem 14
About 35,000 ft elevation, air pressure, reading on mercury manometer is 180 mm Hg. Compute corresponding pressure as Psi and Pa?

(Use: S=0.136, \rho_{mer} = 1009 kg/m^3, \rho = 10 m/s)

\[
\text{Ans: } 2.48 \text{ Psi}
\]

Solution [Psi]:
\[
\rho = gh = \frac{S}{h_0} \rho_{H_2O} h = 13.6 \times 62.4 \text{ lb/ft}^3 \times (0.18) / 0.3048 = 3.48 \text{ Psi}
\]

Solution [Pa]:
\[
P = 13.6 \times \rho_{mer} S \approx 2448 \text{ Pa}
\]

Problem 15
A venturi tube, fabricated with a machined entrance cone is inserted in a 4 in. inside diameter pipe and has a throat diameter of 3 in.
Pressure sensing lines are filled with water and pressure transducer reads 4.2 psi, what is the water flow rate? \rho_{water} = 62.4 lb/ft^3, C=0.995 for machined entrance cone. Use \mu = 2.70 lbm/hr-ft. Reynolds number for steady state flow: 2 \times 10^6 \leq Re \leq 2 \times 10^8.

\[
\text{Ans: } 1.84 \text{ ft}^3/\text{s}
\]

Solution:
\[
Q = \frac{CA_2}{\sqrt{1 - (A_2/A)^2}} \sqrt{\frac{2 \Delta P}{\rho}} = 0.995 \times \frac{\pi (3/2)^2/4}{\sqrt{1 - (3/4)^2}} \sqrt{\frac{2 \times 4.2 \times 144}{62.4}} = 1.84 \text{ ft}^3/\text{s}
\]
\[
V = \frac{Q}{A} = 24.1 \text{ ft/s}
\]
\[
Re \approx 575454 \text{ OK}
\]
### Table 6.6: Student's t as a Function of α and ν

<table>
<thead>
<tr>
<th>ν</th>
<th>0.100</th>
<th>0.050</th>
<th>0.025</th>
<th>0.010</th>
<th>0.005</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.078</td>
<td>6.314</td>
<td>12.706</td>
<td>31.823</td>
<td>63.658</td>
</tr>
<tr>
<td>2</td>
<td>1.886</td>
<td>2.920</td>
<td>4.303</td>
<td>6.964</td>
<td>9.295</td>
</tr>
<tr>
<td>3</td>
<td>1.631</td>
<td>2.353</td>
<td>3.182</td>
<td>4.541</td>
<td>5.841</td>
</tr>
<tr>
<td>4</td>
<td>1.533</td>
<td>2.132</td>
<td>2.776</td>
<td>3.747</td>
<td>4.494</td>
</tr>
<tr>
<td>6</td>
<td>1.476</td>
<td>2.015</td>
<td>2.571</td>
<td>3.365</td>
<td>4.022</td>
</tr>
<tr>
<td>8</td>
<td>1.440</td>
<td>1.943</td>
<td>2.447</td>
<td>3.143</td>
<td>3.707</td>
</tr>
<tr>
<td>10</td>
<td>1.415</td>
<td>1.895</td>
<td>2.365</td>
<td>2.998</td>
<td>3.499</td>
</tr>
<tr>
<td>12</td>
<td>1.397</td>
<td>1.860</td>
<td>2.306</td>
<td>2.896</td>
<td>3.355</td>
</tr>
<tr>
<td>14</td>
<td>1.383</td>
<td>1.833</td>
<td>2.262</td>
<td>2.821</td>
<td>3.250</td>
</tr>
<tr>
<td>16</td>
<td>1.372</td>
<td>1.812</td>
<td>2.229</td>
<td>2.764</td>
<td>3.169</td>
</tr>
<tr>
<td>18</td>
<td>1.363</td>
<td>1.796</td>
<td>2.201</td>
<td>2.718</td>
<td>3.106</td>
</tr>
<tr>
<td>20</td>
<td>1.356</td>
<td>1.782</td>
<td>2.180</td>
<td>2.671</td>
<td>3.054</td>
</tr>
<tr>
<td>24</td>
<td>1.345</td>
<td>1.756</td>
<td>2.145</td>
<td>2.624</td>
<td>3.012</td>
</tr>
<tr>
<td>30</td>
<td>1.341</td>
<td>1.735</td>
<td>2.131</td>
<td>2.592</td>
<td>2.977</td>
</tr>
<tr>
<td>60</td>
<td>1.323</td>
<td>1.645</td>
<td>2.100</td>
<td>2.527</td>
<td>2.897</td>
</tr>
</tbody>
</table>

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### Table 6.7: Critical Values of the Chi-Squared Distribution

<table>
<thead>
<tr>
<th>ν</th>
<th>0.005</th>
<th>0.050</th>
<th>0.250</th>
<th>0.500</th>
<th>0.750</th>
<th>0.950</th>
<th>0.995</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0004</td>
<td>0.0040</td>
<td>0.0550</td>
<td>0.2101</td>
<td>0.6604</td>
<td>2.7005</td>
<td>6.6349</td>
</tr>
<tr>
<td>2</td>
<td>0.0045</td>
<td>0.0208</td>
<td>0.1103</td>
<td>0.3841</td>
<td>0.9236</td>
<td>3.4172</td>
<td>6.2517</td>
</tr>
<tr>
<td>3</td>
<td>0.0114</td>
<td>0.0384</td>
<td>0.1820</td>
<td>0.5705</td>
<td>1.3067</td>
<td>4.1156</td>
<td>6.9485</td>
</tr>
<tr>
<td>4</td>
<td>0.0220</td>
<td>0.0580</td>
<td>0.2679</td>
<td>0.7641</td>
<td>1.7009</td>
<td>4.8384</td>
<td>7.6410</td>
</tr>
<tr>
<td>5</td>
<td>0.0341</td>
<td>0.0796</td>
<td>0.3577</td>
<td>0.9671</td>
<td>2.1034</td>
<td>5.5671</td>
<td>8.3417</td>
</tr>
<tr>
<td>6</td>
<td>0.0484</td>
<td>0.1028</td>
<td>0.4507</td>
<td>1.1782</td>
<td>2.5153</td>
<td>6.2914</td>
<td>9.0484</td>
</tr>
<tr>
<td>7</td>
<td>0.0645</td>
<td>0.1273</td>
<td>0.5466</td>
<td>1.3970</td>
<td>2.9367</td>
<td>6.9984</td>
<td>9.7480</td>
</tr>
<tr>
<td>8</td>
<td>0.0824</td>
<td>0.1532</td>
<td>0.6453</td>
<td>1.6244</td>
<td>3.3678</td>
<td>7.6988</td>
<td>10.4484</td>
</tr>
<tr>
<td>9</td>
<td>0.1019</td>
<td>0.1807</td>
<td>0.7466</td>
<td>1.8594</td>
<td>3.8085</td>
<td>8.3913</td>
<td>11.1484</td>
</tr>
<tr>
<td>10</td>
<td>0.1229</td>
<td>0.2101</td>
<td>0.8505</td>
<td>2.1032</td>
<td>4.2589</td>
<td>9.0772</td>
<td>11.8484</td>
</tr>
<tr>
<td>11</td>
<td>0.1455</td>
<td>0.2414</td>
<td>0.9567</td>
<td>2.3556</td>
<td>4.7190</td>
<td>9.7558</td>
<td>12.5484</td>
</tr>
<tr>
<td>12</td>
<td>0.1697</td>
<td>0.2746</td>
<td>1.0652</td>
<td>2.6166</td>
<td>5.1895</td>
<td>10.4256</td>
<td>13.2484</td>
</tr>
</tbody>
</table>

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\[
S = \sqrt{\sum_{i=1}^{n} \frac{(X_i - \mu)^2}{n(n-1)}} \\
\nu = \frac{\left(\frac{n-1}{\mu} \frac{(n-1)^{-1}}{\sigma^2}\right)}{\nu} \\
P(r) = \left(\frac{n}{r} \right)^{\frac{(n-1)}{2}} \frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} \\
P(x) = \frac{e^{-\lambda x}}{x!} \\
z = \frac{x - \mu}{\sigma} \\
\bar{z} = \frac{x - \bar{z}_2}{\sigma / \sqrt{n}} \\
\sqrt{(n-1)S^2} / \frac{\sigma^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2} = \frac{(n-1)S^2}{\sigma^2} = \frac{x_i - \bar{x}}{\bar{x}} \\
P(x) = \frac{e^{-\lambda x}}{x!} \\
P_{\text{final}} = \int S \cdot W_{\text{full}} = \left(\frac{B_2 + P_2}{2}\right)^{\frac{1}{2}} = \frac{\sqrt{\frac{2\Delta P}{\rho}}}{V} \\
Q = \frac{CA_2}{\sqrt{1 - (A_2/A)'^2}} \\
\text{Re} = \frac{\rho V D}{\mu}
\]