7.1 (a) Series

\[ R = R_1 + R_2 \qquad \frac{\partial R}{\partial R_1} = 1, \quad \frac{\partial R}{\partial R_2} = 1 \]

\[ w_R = [(\frac{\partial R}{\partial R_1} w_{R_1})^2 + (\frac{\partial R}{\partial R_2} w_{R_2})^2]^{1/2} \]

\[ = [(0.2 * 1)^2 + (0.1 * 1)^2]^{1/2} = 0.22 \Omega \]

\[ w_{R,\text{max}} = \left| \frac{\partial R}{\partial R_1} w_{R_1} \right| + \left| \frac{\partial R}{\partial R_2} w_{R_2} \right| \]

\[ = 0.2 + 0.1 = 0.3 \Omega \]

(b) parallel

\[ R = \frac{R_1 R_2}{R_1 + R_2} \qquad \frac{\partial R}{\partial R_1} = \frac{R_2^2}{(R_1 + R_2)^2}, \quad \frac{\partial R}{\partial R_2} = \frac{R_1^2}{(R_1 + R_2)^2} \]

\[ = \frac{50^2}{(150)^2} = \frac{100^2}{150^2} \]

\[ = 0.11 = 0.44 \]

\[ w_R = [(0.11 * 0.2)^2 + (0.44 * 0.1)^2]^{1/2} = 0.05 \Omega \]

\[ w_{R,\text{max}} = 0.11 \times 0.2 + 0.44 \times 0.1 = 0.07 \Omega \]
7.5

\[ \omega = \left( \frac{F}{mr} \right)^{1/2}, \omega^2 = 250,000, \omega = 500 \text{ rad / sec} \]

Eq. 7.6

\[ \frac{w_{\omega}}{\omega} = \frac{1}{2} \left[ \left( \frac{w_m}{m} \right)^2 + \left( \frac{w_r}{r} \right)^2 + \left( \frac{w_F}{F} \right)^2 \right]^{1/2} \]

\[ \frac{w_{\omega}}{\omega} = \frac{1}{2} \left[ \left( \frac{0.5}{100} \right)^2 + \left( \frac{0.02}{20} \right)^2 + \left( \frac{0.5}{500} \right)^2 \right]^{1/2} \]

\[ = \frac{1}{2} \left[ 25 \times 10^{-8} + 10^{-8} + 10^{-8} \right]^{1/2} \]

\[ \frac{w_{\omega}}{\omega} = \pm 2.60 \times 10^{-3}, 0.26\% (95\% \text{ confidence level}) \]

\[ w_{\omega} = \pm 1.30 \text{ rad / sec} (95\% \text{ confidence level}) \]

7.6

\[ E = (f / A)(\frac{\delta L}{L}) = \frac{F}{A.\delta L} \]

Eq. 7.6

\[ \frac{W_E}{E} = \left[ \left( \frac{W_F}{F} \right)^2 + \left( \frac{W_L}{L} \right)^2 + \left( \frac{W_A}{A} \right)^2 + \left( \frac{W_{\delta L}}{\delta L} \right)^2 \right]^{1/2} \]

\[ = \left[ \left( \frac{0.5}{100} \right)^2 + \left( \frac{1}{100} \right)^2 + \left( \frac{1.5}{100} \right)^2 + \left( \frac{5}{100} \right)^2 \right]^{1/2} \]

\[ = \frac{1}{100} [0.25 + 1 + 2.25 + 25]^{1/2} = 5.3 / 100 \text{ or } 5.3\% \]

To reduce the uncertainty in \( E \) by 50\%, we have to reduce the uncertainty in measurement of \( \delta L \) (to less than 1/2 of the present value.)
7.9

hysteresis $\pm 0.1\text{C}$

Lineariz.error $0.2\%$ of reading

Resolution error random

zero off set $0.1\text{C}$

repeatability $\pm 0.2\text{C}$

$B = (0.1^2 + [(0.002)(120)]^2 + 0.1^2)^{1/2}$

$= 0.28\text{C}$

Assuming that the random errors have been determined with samples $> 30$,

$P = (0.05^2 + 0.2^2) = 0.21\text{C}$

So total uncertainty

$w = [B^2 + P^2]^{1/2} = [B^2 + (tS)^2]^{1/2}$

$w = [(0.28)^2 + (0.21)^2]^{1/2}$

$w = 0.35\text{C}$

7.11

accuracy $\pm 0.5\%$ range $= \pm 5 \text{kPa}$

resolution $\pm 1 \text{kPa}$

linearity $\pm 4 \text{kPa}$

temp. stab. $\pm 2 \text{kPa}$ (0 - 50C)

Systematic Errors: Normally, accuracy includes the error of linearity, so the only systematic error is the accuracy. Then, $B = 5 \text{kPa}$.

Random Errors: The random errors are those due to resolution and temperature stability.

Assuming more than 30 samples, $t = 2$

$P = (t^2 + (2)^2)^{1/2} = 2.2 \text{kPa}$

Overall Uncertainty

$w = [B^2 + P^2]^{1/2} = (5^2 + 2.2^2)^{1/2}$

$= 5.5 \text{kPa}$

or $5.5 / 500 = 0.011$ or $1.1\%$ of measured value of $500 \text{kPa}$
7.14 (a) If measurement is performed on over 50 pieces of pipe. Based on problem 7.13, for the sample of 10

\[ m_{av} = \frac{\sum m_i}{n} = 1.94\text{kg} \]

(b) Based on Problem 7.13,

\[ S_{samp} = \left[ \frac{\sum (m_i - m_{av})^2}{n-1} \right]^{1/2} = 1.65 \times 10^{-2} \text{kg} \]

But now, n=50 so the standard deviation of the mean is:

\[ S_{mean} = \frac{S_{samp}}{\sqrt{n}} = \frac{1.65 \times 10^{-2}}{\sqrt{50}} = 0.23 \times 10^{-2} \text{kg} \]

(c) \( P_{single} = tS_{samp} = 2 \times 1.65 \times 10^{-2} \text{kg} = 3.3 \times 10^{-2} \text{kg} \) (95% Confidence)

\( t = 2 \) is for \( \nu > 30 \) degrees of freedom from Student-1 Table 6.6

\( B_{single} = 0.015 \times \text{Range} = 0.015 \times 5\text{kg} = 0.075\text{kg} \)

Total uncertainty: \( w_{single} = \left( P_{single}^2 + B_{single}^2 \right)^{1/2} = 0.082 \) (95% confidence)

(d) \( P_{mean} = tS_{mean} = 2 \times 0.23 \times 10^{-2} = 0.46 \times 10^{-2} \text{kg} \) (95% Confidence)

\( B_{mean} = B_{single} = 0.075\text{kg} \)

Total uncertainty: \( w_{mean} = \left( P_{mean}^2 + B_{mean}^2 \right)^{1/2} = 0.075 \) (95% confidence)

As can be seen, the dominant factor is the systematic uncertainty.
\[ w_{\text{Total}} = \frac{1.5}{100} \times 4.5 = 0.07 \text{ kg} \]

Calib : \[ B_1 = 0.01 \times 4.5 = 0.045 \text{ kg} \]

Calibration will be the only systematic uncertainty.

\[ w_{\text{Total}}^2 = B^2 + P^2 \]

\[ P^2 = w_{\text{Total}}^2 - B^2 \]

\[ w_{\text{Total}} = 0.015 \times 4.5 = 0.068 \]

\[ = 0.068^2 - 0.045^2 \]

\[ P = 0.05 \text{ kg} \]

For a large sample : \[ S_{\text{Total}} = \frac{P}{2} = 0.027 \]

This the acceptable S for the cheese blocks.

The A/D converter contributes to this precision uncertainty but there may be other sources of precision error and we were asked for the total precision uncertainty.
7.22
Accuracy: 0.2 C (systematic error)
average: 250 C
STD Dev: 0.2 C

(a) Random Uncertainty of Mean Value
degrees of freedom 15-1 = 14, 95% confidence.
Table 6.6 ⇒ t = 2.145

\[
P_x = tS_x \sqrt{n} = (2.145)(0.2) \sqrt{14}
\]

= 0.11 C

\[
w = (B^2 + P^2)^{\frac{1}{2}} = (0.2^2 + 0.11^2)^{\frac{1}{2}}
\]

w = 0.23 C overall accuracy of the mean value

(b) Random Uncertainty - Single Reading

\[
P_i = tS_x = (2.145)(0.2) = .43
\]

\[
w = (B^2 + P^2)^{\frac{1}{2}} = (.2^2 + .43^2)^{\frac{1}{2}}
\]

= 0.5 C accuracy of any single measurement

(c) It will have a greater effect on mean since random error is about the same, but for single reading - the effect will not be as much because random error is greater.

7.25 15 measurements - degrees of freedom = 15 - 1 = 14 at 95% confidence.

\[t = 2.145 \text{(Table 6.6)}\]

Precision index \( S_x = 5 kP \)

\[P = ts_x = 2.145 \times 5 = 10.7 \text{ kPa}\]

In Problem 7.11, there are two systematic errors, accuracy and linearity. However, the linearity is normally included in the accuracy so the only systematic error is the accuracy. As a result:

B=5 kPa

Overall uncertainty of the pressure measurements:
total:

\[
w_x = (B^2_x + P^2_x)^{\frac{1}{2}} = (5^2 + 10.7^2)^{\frac{1}{2}}
\]

\[w_x = 11.8 \text{ kPa}\]
7.33

(a) \[ P = \tau \cdot \omega \]
\[ = 2\pi N \cdot \tau = 2\pi \cdot \frac{3000}{60} \cdot 165 \]
\[ = 51,836 \text{ N.m/sec (Watts)} \]

(b) The Standard deviation of power :
\[ S_p = \left[ \left( \frac{\partial P}{\partial \tau} \cdot S_\tau \right)^2 + \left( \frac{\partial P}{\partial \omega} \cdot S_\omega \right)^2 \right]^{1/2} \]
\[ = \left[ (\omega S_\tau)^2 + (\tau S_\omega)^2 \right]^{1/2} \]
\[ = \left[ (2\pi \cdot 50 \cdot 4)^2 + (165 \cdot 2\pi \cdot 5160)^2 \right]^{1/2} \]
\[ = 1,260 \text{ Watts} \]

(c) Random uncertainty of power, considering the large number of rotations:
\[ P_p = 2S_p = 2,520 \text{ Watts} \]

The systematic uncertainty of power :
\[ B_p = \left[ \left( \frac{\partial P}{\partial \omega} \cdot B_\omega \right)^2 + \left( \frac{\partial P}{\partial \tau} \cdot B_\tau \right)^2 \right]^{1/2} \]
\[ = \left[ (\tau B_\omega)^2 + (\omega B_\tau)^2 \right]^{1/2} \]
\[ = \left[ (165 \cdot 2\pi \cdot \frac{5}{60})^2 + (2\pi \cdot \frac{3000}{60} \cdot 0.7)^2 \right]^{1/2} \]
\[ = 236 \text{ Watts} \]

(d) Total Uncertainty of power :
\[ W_p = \left[ B_p^2 + P_p^2 \right]^{1/2} \]
\[ = \left[ (2,520)^2 + (236)^2 \right]^{1/2} \]
\[ = 2,531 \text{ Watts} \]