Problem Set 2

1. Write a Matlab script, which plots the portfolio opportunity set for two risky assets. The script should begin with clearing all variables in memory, and closing previous graphs. Then the program declares the given parameters of the two assets: $\mu_i, \mu_j, \sigma_i, \sigma_j, \rho_{ij}$. Finally, the program must plot the portfolio opportunity set on a fully labeled graph. The name of your script should be OppSetYourName, for example, OppSetLebronJames. The program must contain clear explanations of every command line, so when I read your script, I can easily follow what you are doing. Your script, together with the resulting graph should be submitted to me on Wednesday, July 31, in class.

2. Consider portfolios consisting of two risky assets, $i$ and $j$, with $\mu_i, \mu_j, \sigma_i, \sigma_j, \rho_{ij}$. Find the portfolio with the minimum variance (i.e., the global minimum-variance portfolio). That is, show that the global minimum variance portfolio invests a fraction $\lambda^*$ in asset $i$, where

$$\lambda^* = \frac{\sigma_j^2 - \sigma_i \sigma_j \rho_{ij}}{\sigma_i^2 + \sigma_j^2 - 2\sigma_i \sigma_j \rho_{ij}}.$$  

3. Suppose an investor has mean-variance utility function $v(\mu, \sigma) = \mu - \frac{1}{2} \sigma^2$. She can invest in any portfolio consisting of two risky assets, with random returns $r_i$ and $r_j$.

   (a) Write the asset allocation problem of this investor.

   (b) Derive the optimal portfolio for the investor. Make sure that you also check that the second order conditions are satisfied.

4. Suppose an investor has vNM utility function $u(\cdot)$. She has initial wealth of $w$, and she can invest in any portfolio consisting of two risky assets, with random returns $r_i$ and $r_j$. The joint distribution of the returns is described by the pdf $f(x, y)$.

   (a) Write the asset allocation problem of this investor.

   (b) Discuss the difficulties with actually solving such problem.

5. Calculate the correlation between the return on some asset $i$ and the return on the tangent portfolio $T$. That is, calculate $\text{Corr}(r_i, r_T)$. Express your answer in matrix form.