1. Suppose that the weak preference relation \( \succeq \) on \( L \) is transitive. Prove that the strict preference relation \( \succ \) is transitive.

2. Suppose that the weak preference relation \( \succeq \) on \( L \) is transitive. Prove that the indifference relation \( \sim \) is transitive.

3. Find the certainty equivalents and risk premia of the lottery,

\[
L = \begin{cases} 
$1000 & \text{w.p. } 1/2 \\
$500 & \text{w.p. } 1/2 
\end{cases}
\]

for the following vNM utility functions:

(a) \( u_1(x) = \sqrt{x} \).
(b) \( u_2(x) = \ln(x) \).

4. Consider the gamble in St. Petersburg Paradox paradox example, with payoffs \( 2^{k-1} \), \( k = 1, 2, \ldots \), and the associated probabilities \( (\frac{1}{2})^k \), \( k = 1, 2, \ldots \). Suppose preferences over lotteries can be represented with expected utility form, with vNM functional \( u(x) \). Calculate the expected utility of the gamble, and its certainty equivalent for the following vNM utility functions:

(a) \( u(x) = \ln(x) \).
(b) \( u(x) = \sqrt{x} \).
(c) Explain why individual with vNM utility function \( u(x) = \ln(x) \) is willing to pay less for the gamble than individual with vNM utility function \( u(x) = \sqrt{x} \).

5. Consider two lotteries:

\[
X \sim U [0, 2] \quad \text{and} \quad Y = \begin{cases} 
1.2475 & \text{w.p. } 0.8 \\
0.01 & \text{w.p. } 0.2 
\end{cases}
\]

This means that \( X \) has a continuous uniform distribution on the interval \([0, 2] \).

(a) Calculate the mean and variance of both lotteries.
(b) Which lottery is preferred by an individual whose vNM utility function is \( u(x) = \ln(x) \)?
(c) Based on your answers to (b) and (c), is it true that any risk averse individual, when comparing two lotteries with the same mean, would always prefer the one with the lower variance?