1 Introduction

Recall from the introduction that the output per capita in the U.S. is growing steady, but there are fluctuations about the trend. These fluctuations are called business cycles. Figure 1 shows the ln of real GNP per capita in the U.S. in the last century, together with a linear trend. The linear trend fits the data pretty well, which means that the original variable, GNP per capita, was growing at constant rate.

The study of the long run growth trend belongs to the field of economic growth. In these notes we focus on the fluctuations of the output around the trend. Subtracting the growth trend from the time series in figure 1, results in a series of deviations from trend, displayed in figure 2. The series of deviations from trend is called detrended real output, or the cyclical part of the real output.

The questions that we want to ask in these notes are:

1. What causes business cycles?
2. Can the government smooth out the business cycles?
3. Should the government smooth out the business cycles?

In order to answer these questions, economists use models. We will see that different models give different answers to those questions.
2 The Classical Model

2.1 The description of the model

The model consists of a representative consumer, representative firm, and a government. The consumer receives income from supplying his labor and from dividends from the firm he owns. The consumer chooses his consumption and time allocation between labor and leisure. The firm is owned by the consumer, it owns a fixed amount of capital, and it chooses the optimal amount of labor to maximize profits. The government consumption is exogenous to the model. The government balances its budget by collecting taxes at the amount of expenditures.

The formal description of the model economy:

1. Consumer:

\[
\max_{C,l} \alpha \ln C + (1 - \alpha) \ln l \\
\text{s.t.} \\
C = [w(h - l) + \pi](1 - t)
\]

where \(C\) is consumption, \(l\) is leisure, \(w\) is real wage, \(h\) is time endowment (say 100 hours per week), \(\pi\) is the profits or dividends from the firm, \(t\) is the flat tax rate. Thus, the time spent working (labor supply) is

\[L_S = h - l\]

2. Firm:

\[
\max_{L_D} \pi = AK^\theta L_D^{1-\theta} - wL_D
\]
where $A$ is productivity parameter (called Total Factor Productivity, TFP), $K$ is the capital stock, and $L_D$ is labor employed by the firm. The productivity parameter reflects the idea that with technological improvement ($A \uparrow$) more output can be produced with the same inputs. The total output in the economy is thus $Y = AK^\theta L^{1-\theta}$.

3. **Government**: collects taxes on all income at the rate of $t$, and spends them on government consumption. The government budget is

$$G = t\left(wL + \pi\right)$$

4. **Definition**: Competitive equilibrium consists of $(w, C, G, L, l, \pi, Y)$ such that

(a) Given $w, \pi$, the values of $(C, l)$ solve the consumer’s problem,

(b) Given $w$, the value of $L$ solves the firm’s problem,

(c) Markets are cleared:

i. $L_D = L_S = L$ (labor market),

ii. $C + G = Y$ (final goods market).

### 2.2 Important remarks about models in general

This section is a philosophical discussion of our approach in general. It is essential to read it in order to understand the material of this entire course, and many other courses that you are taking. You should come back and read this again after you have practiced working with the classical model.

1. **The competitive equilibrium** is the model’s prediction about the endogenous variables. Endogenous variables are determined inside the model, i.e. the variables which the model is trying to explain. The exogenous variables are those that are determined outside of the model. For example, in the model of a market the endogenous variables are price and quantity traded, and exogenous variables are those that determine the location of supply and demand curves (such as income, prices of related goods, etc.). In the classical model the exogenous variables are: $(A, t, K)$, and the endogenous variables are: $(w, C, G, L, l, \pi, Y)$.

2. **Causality: what causes what?** In any model, the exogenous variables are “causing” the endogenous variables. For example, we can change $A$ (the technology level) and observe the changes in real wage, employment, consumption, output, etc. All the endogenous variables are caused by the exogenous variables. If we don’t change any of the exogenous variables, no change in the endogenous variables can occur. Thus, in this model we cannot say that “output causes employment”, since both output and employment are endogenous variables, and cannot change unless we change some of the exogenous. Be very careful about making statements of causality in the real world.

3. **Why models?**
(a) Models can explain some features of the real world. Models don’t tell us what the world looks like. Instead, they tell us what we can expect to happen in the world if the world was like the model. For example, the supply and demand diagram doesn’t look anything like the markets in the real world. The diagram does not show the identities of the buyers and sellers, their feelings and emotions, their physical appearance. The supply and demand diagram only captures two features of real markets: (1) buyers typically want to buy less when the price goes up, and (2) sellers want to produce more when the price goes up. It turns out the supply and demand diagram is very useful in explaining why prices differ across goods and why there are changes in prices. After testing the predictions of the model with the data we conclude that indeed the two features of buyers and sellers that we included in the model were important.

(b) Models can be used to perform controlled experiments. In the real world many things change at the same time; the technology changes, government policies change, etc. In the model we can perform controlled experiments of changing one thing at a time. This is impossible to do with actual economies.

4. **Models are not realistic and are not supposed to be.** When the object of study is very complicated, we need models that will highlight some important features of the object and leave out many other features. For example, when we study the economy of an entire country with millions of people, thousands of markets and firms, it is difficult for us to understand the behavior of the economy by just looking at it. Moreover, if we don’t have any models to work with, we don’t even know what data should be collected about the object of our study. For example, the model of supply and demand tells us that we don’t need to collect data of all the names of buyers and sellers of the market in order to understand how it works.

2.3 Working with the classical model

The definition of competitive equilibrium is instructive about how the model should be solved. The definition suggests the following steps: (1) solve the consumer’s problem to get the labor supply, (2) solve the firm’s problem to get the labor demand, and (3) use the market clearing conditions to find the real wage, the equilibrium employment, and the rest of the endogenous variables.

2.3.1 Mathematical solution

**Step 1: solving the consumer problem**

The consumer’s problem can be written as

\[
\max_{C,l} \alpha \ln C + (1 - \alpha) \ln l
\]

\[
s.t.
\]

\[
C + w(1 - t)l = (wh + \pi)(1 - t)
\]
This is a standard consumer choice problem with two goods: $C$ and $l$, the prices of the goods are $1$ and $w(1 - t)$ respectively, and the consumer’s income is $(wh + \pi)(1 - t)$. We know already how to solve a consumer choice problem with Cobb-Douglas preferences. Thus, the demand is

$$C = \alpha (wh + \pi)(1 - t)$$
$$l = (1 - \alpha) \frac{(wh + \pi)(1 - t)}{w(1 - t)} = (1 - \alpha) \left( h + \frac{\pi}{w} \right)$$

and the labor supply is

$$L_S = h - (1 - \alpha) \left( h + \frac{\pi}{w} \right)$$ \hspace{1cm} (1)

Observe that consumption is increasing in $w$ and $\pi$, and decreases in $t$. The labor supply is increasing in $w$, decreasing in $\pi$ and does not depend on taxes. The intuition why the labor supply is decreasing in $\pi$ goes as follows. The dividend income is non labor income, so when it goes up the consumer does not need to work as much. Figure 3 shows the graph of the labor supply curve. i.e. how much labor the consumer wants to supply at any given wage, holding everything else fixed. This means that changes in $w$ are reflected by movements along the curve, while changes in $\pi$ will shift the entire curve.

![Labor supply curve](image)

**Step 2: Solving the firm’s problem**

The firm’s problem is

$$\max_{L_D} \pi = AK^\theta L_D^{1-\theta} - wL_D$$

The first order condition

$$\frac{\partial \pi}{\partial L_D} = (1 - \theta) AK^\theta L_D^{-\theta} - w = 0$$

$$(1 - \theta) AK^\theta L_D^{-\theta} = w$$ \hspace{1cm} (2)
which tells us that the firm maximizes profit when it equates the marginal product of labor the the real wage. Equation (2) thus gives us the labor demand of the firm. We can solve for $L_D$ explicitly from equation (2) to get

$$L_D = \left( \frac{(1 - \theta) AK^\theta}{w} \right)^{1/\theta}$$

Observe that this curve is decreasing in $w$. Figure 4 shows the labor demand curve, i.e. how much labor the firm wants to employ at any given wage, holding everything else constant. Thus, changes in $w$ are reflected by movements along the curve while changes in $A$ will shift the entire curve. The profit is therefore given by

$$\pi = AK^\theta L_D^{1-\theta} - (1 - \theta) AK^\theta L_D^{-\theta} \cdot L_D = \theta AK^\theta L_D^{1-\theta}$$

(3)

**Step 3: equilibrium in the labor market**

Letting $L_S = L_D = L$ and substituting equations (2) and (3) into equation (1) gives

$$L = h - (1 - \alpha) \left( h + \frac{\theta AK^\theta L^{1-\theta}}{(1 - \theta) AK^\theta L^{-\theta}} \right)$$

Solving for equilibrium $L$:

$$L = h - (1 - \alpha) \left( h + \frac{\theta}{(1 - \theta) \theta L} \right)$$

$$L = h - (1 - \alpha) h - \frac{(1 - \alpha) \theta}{(1 - \theta) L}$$


\[
L + \frac{(1 - \alpha)\theta}{(1 - \theta)} L = \alpha h \\
L \left[ 1 + \frac{(1 - \alpha)\theta}{1 - \theta} \right] = \alpha h \\
L \left[ 1 - \theta + \frac{(1 - \alpha)\theta}{1 - \theta} \right] = \alpha h \\
L \left[ \frac{1 - \alpha \theta}{1 - \theta} \right] = \alpha h
\]

Equilibrium employment: \( L^* = \frac{\alpha (1 - \theta) h}{1 - \alpha \theta} \)

Once we found the equilibrium employment \( L^* \), all the other endogenous variables can be found in terms of \( L^* \). Equilibrium leisure:

\( l^* = h - L^* \)

To solve for equilibrium wage, use equation (2):

\( w^* = (1 - \theta) AK^\theta L^{* - \theta} \)

Equilibrium output:

\( Y^* = AK^\theta L^{*1 - \theta} \)

Equilibrium profit, using equation (3):

\( \pi^* = \theta AK^\theta L^{*1 - \theta} \)

To find equilibrium consumption we use the budget constraint:

\[
C^* = \left[ w^* L^* + \pi^* \right] (1 - t) \\
C^* = \left[ (1 - \theta) AK^\theta L^{*1 - \theta} + \theta AK^\theta L^{1 - \theta}_D \right] (1 - t) \\
C^* = (1 - t) Y^*
\]

Equilibrium government expenditures:

\( G^* = Y^* - C^* = tY^* \)

**Summary of equilibrium:**

\[
L^* = \frac{\alpha (1 - \theta) h}{1 - \alpha \theta} \\
l^* = h - L^* \\
w^* = (1 - \theta) AK^\theta L^{* - \theta} \\
Y^* = AK^\theta L^{*1 - \theta} \\
\pi^* = \theta AK^\theta L^{*1 - \theta} \\
C^* = Y^* (1 - t) \\
G^* = tY^*
\]
As you can see, an increase in productivity $A$, causes an increase in equilibrium output, equilibrium real wage, equilibrium consumption, equilibrium government consumption, and equilibrium profit. Equilibrium employment does not depend on the level of technology, even though the real wage went up. The effect of higher $K$ is similar because $A$ and $K$ always appear together in the equations.

An increase in the tax rate affects only the distribution of the total output between the private sector and the government sector. If $t = 30\%$ for example, then the government consumes $30\%$ of the total output, while the private consumers get to consume the rest $70\%$. 
2.3.2 Graphical analysis

The classical model can be analyzed graphically with only two diagrams, the labor market and the production function, as shown in figure 5.

These graphs correspond to the following equations:

Production function :  \( Y = AK^\theta L^{1-\theta} \)

Labor supply curve :  \( L_S = h - (1 - \alpha) \left( h + \frac{\pi}{w} \right) \), where \( \pi = \theta AK^\theta L^{1-\theta} \)

Labor demand curve :  \( L_D = \left( \frac{(1 - \theta) AK^\theta}{w} \right)^{1/\theta} \)

It is important to repeat here that labor supply curve is increasing in \( w \). On the other hand, if \( \pi \) increases, this leads to a shift of the entire supply curve to the left. The labor demand curve is decreasing in \( w \). If \( A \) or \( K \) increase, the entire labor demand curve will shift to the right.

Now we use this graphical framework in order to perform 3 experiments with the model:

1. An increase in productivity (\( A \uparrow \)).
Figure 6 shows the effects of an increase in $A$ in the classical model.

As $A \uparrow$, there is an increase in labor demand (shift of the labor demand curve to the right) and a decrease in labor supply (shift of the supply curve to the left) and an increase in production function. The effect of the increase in labor demand on employment is an increase in employment, while the effect of a decrease in labor supply on employment is a decrease in employment. Thus, without solving the model with particular functional forms we cannot tell what is the effect of $A \uparrow$ on equilibrium employment. In the previous section however we solved the model with Cobb-Douglas technology and preferences and found that the equilibrium employment does not change as $A \uparrow$. In other words, the effect on employment of a decline in labor demand and of an increase in labor supply cancel each other. Both effects however increase the equilibrium real wage.

2. An increase in $K$.

The effect of an increase in $K$ is the same as the effect of an increase in $A$. Notice that $A$ and $K$ always appear together as $AK^\theta$.

3. An increase in the tax rate ($t \uparrow$).
Neither the labor demand nor the labor supply depend on the tax rate, hence nothing will change in the labor market. The production function does not depend on the tax rate as well, and therefore none of the curves in figure 5 will shift. As we have seen before, the only effect that an increase in the tax rate has on the economy is the increase in the government share of the total output.

2.3.3 Answering the questions

Now we are ready to answer the questions we posed in the beginning of these notes, within the framework of the classical model.

1. What causes business cycles?

The exogenous variables in this model are \((A, t, K)\). As we have seen before, a positive shock to productivity \((A \uparrow)\) increases the equilibrium output while a negative shock to productivity \((A \downarrow)\) decreases it. We can think of shocks to productivity as agglomeration of many factors such as innovations, shocks to oil prices, weather, political events, etc., that change the amount produced with the same inputs\(^1\). So this model suggests that business cycles might be a result of productivity shocks. As we have seen before, changes in \(t\) do not affect the equilibrium output. How about \(K\)? It is possible that a hurricane, or a terrorist attack would destroy part of the nation’s capital and cause a decline in output. It is harder to think of how the stock of capital can experience a sudden increase. In any case, when we look at the data on capital stock, it looks very smooth and does not exhibit fluctuations that can potentially be the cause of business cycles. Moreover, in applied versions of this model, \(K\) is endogenous (accumulated within the model through investment), and therefore, in such models we can’t exogenously change \(K\).

2. Can the government smooth out the business cycles?

We have seen before that changes in the tax rate in this economy do not affect the equilibrium output. Changes in the tax rate only affect the fraction of total output that is consumed by the government. Recall that

\[
C = (1 - t)Y \\
G = tY
\]

So the answer to the question is, NO, the government cannot smooth the business cycles (in this model).

3. Should the government smooth out the business cycles?

It shouldn’t because it can’t (in this model).

\(^1\)For a more detailed discussion about productivity shocks see the next section.
2.4  Real business cycle doctrine

Real business cycle theory suggests that the main source of business cycles is shocks to productivity. The real business cycle school is led by Edward Prescott and Finn Kydland. They were awarded a Nobel Prize in Economics in economics in 2004 "for their contributions to dynamic macroeconomics: the time consistency of economic policy and the driving forces behind business cycles". Finn Kydland and Edward Prescott developed a methodology that allows them to answer the following quantitative equation: "how much of the fluctuations in output around a trend can be accounted for by random shocks to productivity?". Their answer was 2/3. Kydland and Prescott used a model that is a more complex version of the classical model (their model is called "the Neoclassical Growth Model"). But the idea can be illustrated with the classical model.

**Step 1: Choose functional forms for utility and production function.**

In the data, although the real wage went up in the last decades, the average worktime did not change. Notice that in our model with Cobb-Douglas utility function, we get the same result, i.e. in equilibrium the worktime is constant and does not depend on the real wage.

In the data, the labor share of total output is roughly constant over time. The Cobb-Douglas production function delivers this property. Recall that the capital share is \( \theta \) and the labor share is \( 1 - \theta \), and these are constant.

**Step 2: Choose the parameter values for the utility and production functions.**

In the data the labor share is about 2/3 of the total output. Thus, set \( \theta = 1/3 \) so that \( 1 - \theta = 2/3 \). In the real world people have approximately 100 hours per week that they can allocate between labor and leisure activity (24 hours per day, minus 8 hours of sleep and 2 hours of maintenance such as bathroom, eating, resting). In the data the average worktime is 40 hours per week, so using our equilibrium equation for employment we can find \( \alpha \) as follows:

\[
L^* = \frac{\alpha (1 - \theta) h}{1 - \alpha \theta} \\
40 = \frac{\alpha (1 - \frac{1}{3}) 100}{1 - \alpha \frac{1}{3}} \\
0.4 = \frac{\alpha^2}{1 - \alpha \frac{1}{3}} \\
0.4 - \alpha \cdot 0.4 \cdot \frac{1}{3} = \frac{2}{3} \\
1.2 - 0.4 \alpha = 2a \\
1.2 = 2.4a \\
\alpha = 0.5
\]

Thus, \( \alpha = 0.5, \theta = \frac{1}{3} \).

**Step 3: Estimate the shocks to productivity**

We assume that aggregate output is produced with

\[
Y = AK^\theta L^{1-\theta}
\]
We have data on real GDP ($Y$), on capital ($K$) and labor employed ($L$). This means that we can find $A$ from the above equation as a residual. Because of this procedure $A$ is called the Solow Residual, since we find it as the residual that would equate the left hand side and the right hand side of equation (4).

**Step 4: Model simulation**

Having found the time series of $A$ we can simulate the model and generate time series of consumption and output. We have seen that an increase in $A$ causes an increase in output in the classical model and a decline in $A$ will cause a decline in output. It turns out that the time series of $Y$ generated by the model is very similar to the data in figure 2. In fact, the variance of the output generated by the model is about $2/3$ of the actual variance of the real GDP/capita in the data. This means that random shocks to productivity can explain most, but not all the variation in real GDP/capita over the business cycles.