Midterm Exam*

Wednesday, October 29

50 minutes

Name: ________________________________

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck 😊
1. (30 points). Consider the Classical model studied in class, and briefly described as follows. The consumer derives utility from consumption $C$ and leisure $l$ according to $U(C,l) = \alpha \ln C + (1-\alpha) \ln l$. He is endowed with $h$ hours which he can allocate between leisure and work $L_s$. The real wage is $w$. The consumer owns a firm and receives dividend income (profit) $\pi$. The firm produces output $Y$ using technology $Y = AK^\theta L_D^{1-\theta}$, where $A$ is productivity parameter (TFP), $K$ is the capital owned by the firm, and $L_D$ is labor employed by the firm. The government taxes labor income at the rate of $t_w$ and dividend income at the rate of $t_\pi$.

a. (5 points). Write the consumer’s problem.

\[
\text{Consumer’s problem} \\
\max_{C,l} \alpha \ln C + (1-\alpha) \ln l \\
\text{s.t.} \\
C = w(h-l)(1-t_w) + \pi(1-t_\pi)
\]

b. (5 points). Write the consumer's demand for consumption.

For finding the demand it is convenient to rewrite the budget constraint as follows:

\[C + w(1-t_w)l = wh(1-t_w) + \pi(1-t_\pi)\]

Demand for consumption: $C = \alpha[wh(1-t_w) + \pi(1-t_\pi)]$

c. (5 points). Write the consumer's demand for leisure and his labor supply.

Demand for leisure: $l = \frac{(1-\alpha)[wh(1-t_w) + \pi(1-t_\pi)]}{w(1-t_w)} = (1-\alpha)\left(h + \frac{\pi(1-t_\pi)}{w(1-t_w)}\right)$

Labor supply: $L_s = h - l = h - (1-\alpha)\left(h + \frac{\pi(1-t_\pi)}{w(1-t_w)}\right)$
d. (5 points). In this model, changes in productivity do not affect the equilibrium labor. True/False, circle the correct answer and provide a brief mathematical proof.

Plugging the profit and wage from the firm's problem into the labor supply, we get

\[ L = h - (1 - \alpha) \left( \frac{\theta AK^{\theta} L^{1-\theta}}{(1 - \theta) AK^{\theta} L^{-\theta}} \frac{(1 - t_{\pi})}{(1 - t_{w})} \right) \]

\[ = h - (1 - \alpha) \left( \frac{\theta}{1 - \theta} \frac{L (1 - t_{\pi})}{(1 - t_{w})} \right) \]

Thus, the productivity \((A)\) cancels out and labor supply is independent of productivity.
e. (10 points). Using fully labeled graphs of the production function and labor market, illustrate the effect of productivity growth ($A \uparrow$) on equilibrium output ($Y^*$), equilibrium real wage ($w^*$) and equilibrium employment ($L^*$).
2. (15 points). Consider the Keynesian model discussed in class. Suppose that the economy is characterized by the following behavioral functions:

Consumption: \[ C = 100 + 0.5(Y - T) \]
Investment: \[ I = 200 \]
Government spending: \[ G = 500 \]
Taxes: \[ T = 100 + 0.5Y \]
Full employment output: \[ Y_f = 1200 \]

a. Solve for the Keynesian equilibrium in the goods market.

\[
Y = E \\
Y = 100 + 0.5(Y - 100 - 0.5Y) + 200 + 500 \\
Y = 100 - 0.5 \cdot 100 + 0.5 \cdot (1 - 0.5)Y + 200 + 500 \\
Y^* = \frac{50 + 200 + 500}{1 - 0.5 \cdot 0.5} = \frac{750}{0.75} = 1000
\]

b. Find the government deficit in equilibrium.

\[ \text{Def} = G - T = 500 - (100 + 0.5 \cdot 1000) = -100 \]

(This means that the government is running a budget surplus of 100).
a. On a fully labeled graph Illustrate the Keynesian equilibrium in the goods market before and after a decrease in proportional tax rate. No numbers are required.
3. (15 points). The following table contains data from the labor market of some country (in millions).

<table>
<thead>
<tr>
<th>Civilian noninstitutional population</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civilian labor force</td>
<td>70</td>
</tr>
<tr>
<td>Employed</td>
<td>63</td>
</tr>
<tr>
<td>Unemployed</td>
<td>7</td>
</tr>
<tr>
<td>Not in the labor force</td>
<td>30</td>
</tr>
</tbody>
</table>

a. Complete the above table.

b. Find the unemployment rate in this country.

\[
\text{Unemp. Rate} = \frac{\text{Unemployed}}{\text{Civilian labor force}} = \frac{7}{70} = 0.1 = 10\%
\]

c. Find the labor force participation rate in this country.

\[
\text{Labor Force Participation rate} = \frac{\text{Civilian labor force}}{\text{Civilian noninstitutional population}} = \frac{70}{100} = 0.7 = 70\%
\]
4. (15 points). Consider the search model of unemployment, briefly described as follows.

<table>
<thead>
<tr>
<th>Fraction in population</th>
<th>Unemployed</th>
<th>Employed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( U )</td>
<td>( 1 - U )</td>
</tr>
<tr>
<td>Utility</td>
<td>( V_u(b, p, t_b) )</td>
<td>( V_e(w, s, t_w) )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
V_u(b, p, t_b) &= b - \text{unemployment insurance benefit} \\
p &= \text{probability of receiving a job offer} \\
t_b &= \text{tax on } b \\
V_e(w, s, t_w) &= w - \text{real wage} \\
s &= \text{separation rate (probability of loosing the job)} \\
t_w &= \text{tax on } w
\end{align*}
\]

The symbols “+” under variable of the utility function indicates the assumption that the utility is increasing in that variable, and “−” under a variable indicates that the utility is decreasing in that variable.

**Distribution of wage offers:** \( H(w) \) gives the probability that an offer is at least \( w \).

Illustrate with 3 fully labeled graphs the impact of an increase in the probability of receiving a job offer (\( p \uparrow \)) on: (1) reservation wage \( w^* \), (2) probability of acceptance of job offers \( H(w^*) \), and (3) steady-state unemployment rate \( U^* \).
Thus, we don’t know what happens to equilibrium unemployment rate: $U_2^*$?
5. (10 points). Suppose that in some economy the private saving is 50, the domestic investment is 50, and the trade deficit is 17. What must be the government budget deficit in that country? Show your calculations.

\[ S_P + S_G = I + NX \]

\[ \frac{50}{50} + S_G = \frac{50}{50} + (-17) \]

\[ S_G = -17 \]

Government budget deficit is 17.