Midterm Exam

Wednesday, October 29

50 minutes

Name: ________________________________

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck 😊
1. (30 points). Consider the Classical model studied in class, and briefly described as follows. The consumer derives utility from consumption $C$ and leisure $l$ according to $U(C, l) = \alpha \ln C + (1 - \alpha) \ln l$. He is endowed with $h$ hours which he can allocate between leisure and work $L_S$. The real wage is $w$. The consumer owns a firm and receives dividend income (profit) $\pi$. The firm produces output $Y$ using technology $Y = AK^\theta L_D^{1-\theta}$, where $A$ is productivity parameter (TFP), $K$ is the capital owned by the firm, and $L_D$ is labor employed by the firm. The government taxes labor income at the rate of $t_w$ and dividend income at the rate of $t_\pi$.

a. (5 points). Write the consumer’s problem.

Consumer’s problem
max \( \alpha \ln C + (1 - \alpha) \ln l \)

s.t.
\( C = w(h - l)(1 - t_w) + \pi(1 - t_\pi) \)

b. (5 points). Write the consumer's demand for consumption.

For finding the demand it is convenient to rewrite the budget constraint as follows:

\( C + w(1 - t_w)l = wh(1 - t_w) + \pi(1 - t_\pi) \)

Demand for consumption: $C = \alpha [wh(1 - t_w) + \pi(1 - t_\pi)]$

c. (5 points). Write the consumer's demand for leisure and his labor supply.

Demand for leisure: $l = \frac{(1 - \alpha)[wh(1 - t_w) + \pi(1 - t_\pi)]}{w(1 - t_w)} = (1 - \alpha) \left( h + \frac{\pi (1 - t_\pi)}{w (1 - t_w)} \right)$

Labor supply: $L_S = h - l = h - (1 - \alpha) \left( h + \frac{\pi (1 - t_\pi)}{w (1 - t_w)} \right)$
d. (5 points). If the government taxes both types of income at the same rate (i.e. $t_w = t_{\pi} = t$), then the tax rate does not affect the labor supply.  

(True/False, circle the correct answer and provide a short mathematical proof.

Labor supply from last section, when $t_w = t_{\pi} = t$ becomes:

$$L_S = h - (1 - \alpha) \left( h + \frac{\pi}{w (1-t)} \right) = h - (1 - \alpha) \left( h + \frac{\pi}{w} \right)$$

Thus, the taxes cancel out and labor supply is independent of taxes.
(10 points). Using fully labeled graphs of the production function and labor market, illustrate the effect of productivity growth \((A \uparrow)\) on equilibrium output \((Y^*)\), equilibrium real wage \((w^*)\) and equilibrium employment \((L^*)\).
2. (20 points). Consider the Keynesian model discussed in class. Suppose that the economy is characterized by the following behavioral functions:

\begin{align*}
\text{Consumption: } & \quad C = 100 + 0.5(Y - T) \\
\text{Investment: } & \quad I = 200 \\
\text{Government spending: } & \quad G = 500 \\
\text{Taxes: } & \quad T = 100 + 0.5Y \\
\text{Full employment output: } & \quad Y_f = 1200
\end{align*}

a. Solve for the Keynesian equilibrium in the goods market.

\begin{align*}
Y &= E \\
Y &= 100 + 0.5(Y - 100 - 0.5Y) + 200 + 500 \\
Y &= 100 - 0.5 \cdot 100 + 0.5 \cdot (1 - 0.5)Y + 200 + 500 \\
Y^* &= \frac{50 + 200 + 500}{1 - 0.5 \cdot 0.5} = \frac{750}{0.75} = 1000
\end{align*}

b. Find the government deficit in equilibrium.

\begin{align*}
\text{Def} &= G - T = 500 - (100 + 0.5 \cdot 1000) = -100
\end{align*}

(This means that the government is running a budget surplus of 100).
c. On a fully labeled graph Illustrate the Keynesian equilibrium in the goods market before and after a fall in investment. No numbers are required.
3. (15 points). The following table contains data from the labor market of some country (in millions).

<table>
<thead>
<tr>
<th>Civilian noninstitutional population</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civilian labor force</td>
<td>60</td>
</tr>
<tr>
<td>Employed</td>
<td>54</td>
</tr>
<tr>
<td>Unemployed</td>
<td>6</td>
</tr>
<tr>
<td>Not in the labor force</td>
<td>40</td>
</tr>
</tbody>
</table>

a. Complete the above table.

b. Find the unemployment rate in this country.

\[
\text{Unemp. Rate} = \frac{\#\text{Unemployed}}{\#\text{Labor Force}} = \frac{6}{60} = 0.1 = 10\% 
\]

c. Find the labor force participation rate in this country.

\[
\text{Labor Force Participation rate} = \frac{\#\text{Labor Force}}{\#\text{Civilian Noninst. Pop.}} = \frac{60}{100} = 0.6 = 60\% 
\]
4. (15 points). Consider the search model of unemployment, briefly described as follows.

<table>
<thead>
<tr>
<th>Fraction in population</th>
<th>Unemployed</th>
<th>Employed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U$</td>
<td>$1 - U$</td>
</tr>
<tr>
<td>Utility</td>
<td>$V_u(b, p, t_b)$</td>
<td>$V_e(w, s, t_w)$</td>
</tr>
<tr>
<td>$b$ – unemployment insurance benefit</td>
<td>$w$ – real wage</td>
<td></td>
</tr>
<tr>
<td>$p$ – probability of receiving a job offer</td>
<td>$s$ – separation rate (probability of loosing the job)</td>
<td></td>
</tr>
<tr>
<td>$t_b$ – tax on $b$</td>
<td>$t_w$ – tax on $w$</td>
<td></td>
</tr>
</tbody>
</table>

The symbols “+” under variable of the utility function indicates the assumption that the utility is increasing in that variable, and “−” under a variable indicates that the utility is decreasing in that variable.

**Distribution of wage offers:** $H(w)$ gives the probability that an offer is at least $w$.

Illustrate with 3 fully labeled graphs the impact of an increase in separation rate ($s \uparrow$) on: (1) reservation wage $w^*$, (2) probability of acceptance of job offers $H(w^*)$, and (3) steady-state unemployment rate $U^*$. 
5. (10 points). Suppose that in some economy the private saving is 100, the domestic investment is 100, and the trade deficit is 17. What must be the government budget deficit in that country? Show your calculations.

\[
\frac{S_P}{100} + S_G = \frac{I}{100} + \frac{NX}{-17}
\]

\[
S_G = -17
\]

Government budget deficit is 17.