Midterm Exam 2

Tuesday, October 28

1 hour and 15 minutes

Name: ___________________________________

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.
1. (50 points). Consider the Classical model studied in class, and briefly described as follows. The consumer derives utility from consumption $C$ and leisure $l$ according to $U(C,l) = \alpha \ln C + (1-\alpha) \ln l$. He is endowed with $h$ hours which he can allocate between leisure and work $L_S$. The real wage is $w$. The consumer owns a firm and receives dividend income (profit) $\pi$. The firm produces output $Y$ using technology $Y = AK^{\theta} L_D^{1-\theta}$, where $A$ is productivity parameter (TFP), $K$ is the capital owned by the firm, and $L_D$ is labor employed by the firm. The government taxes labor income at the rate of $t_w$ and dividend income at the rate of $t_\pi$.

a. Write the consumer’s utility maximization problem.

\[
\text{Consumer’s problem} \\
\max_{C,l} \alpha \ln C + (1-\alpha) \ln l \\
\text{s.t.} \\
C = w(h-l)(1-t_w) + \pi(1-t_\pi)
\]

b. Derive the consumer's demand for consumption and leisure using the results from Microfoundations. Explain your steps briefly.

We rewrite the budget constraint in the form of $p_x x + p_y y = I$:

\[
C + w(1-t_w)l = wh(1-t_w) + \pi(1-t_\pi)
\]

**Explanation:** From Microfoundations, we know that consumers with these Cobb-Douglas preferences, spend a fraction $\alpha$ of their income on $C$ and a fraction $1-\alpha$ of their income on $l$. Thus, the demand is:

Demand for consumption: $C^* = \alpha[wh(1-t_w) + \pi(1-t_\pi)]$

Demand for leisure: $l^* = \frac{(1-\alpha)[wh(1-t_w) + \pi(1-t_\pi)]}{wh(1-t_w)} = (1-\alpha) \left( \frac{h + \frac{\pi}{w} (1-t_\pi)}{w(1-t_w)} \right)$
c. Based on your results from the previous section, write the consumer’s labor supply.

\[ L_S = h - l = h - \left(1 - \alpha\right) \left(h + \frac{\pi}{w} \frac{(1 - t_{\pi})}{(1 - t_w)}\right) \]

Labor supply: or

\[ L_S = \alpha h - \left(1 - \alpha\right) \frac{\pi}{w} \frac{(1 - t_{\pi})}{(1 - t_w)} \]

d. Illustrate graphically the impact on the labor supply of an increase in the tax on dividend income (\( t_{\pi} \uparrow \)), and provide economic intuition for it.

Lower non-labor income (after taxes), increases the incentives to work and earn labor income, so the labor supplied at any given wage decreases.
e. Consider the special case, in which both labor and non-labor income are taxed at the same rate: \( t_w = t_\pi = t \). Illustrate graphically the impact on the labor supply of an increase in the tax rate (\( t \uparrow \)).

With \( t_w = t_\pi = t \), the labor supply is \( L_S = \alpha h - (1 - \alpha) \frac{\pi}{w} \), and it is unaffected by taxes. Thus, there will be no change in the graph of labor supply when \( t \) changes.

\[
\begin{align*}
\text{f. Write the firm’s profit maximization problem.} \\
\max_{L_D} \pi = AK^\theta L_D^{1-\theta} - wL_D
\end{align*}
\]
g. Solve for the firm’s demand for labor and the firm’s profit.

The first order necessary condition for profit maximization:
\[
\frac{\partial}{\partial L_D} \pi = (1 - \theta)AK^\theta L_D^{-\theta} - w = 0
\]

Thus,
\[
(1 - \theta)AK^\theta L_D^{-\theta} = w
\]
\[
L_D = \left( \frac{(1 - \theta)AK^\theta}{w} \right)^{1/\theta}
\]

The profit:
\[
\pi = AK^\theta L_D^{1-\theta} - (1 - \theta)AK^\theta L_D L_D = \theta AK^\theta L_D^{1-\theta}
\]

h. Solve for the equilibrium employment, \( L^* \), in an economy with \( t_w = t_\pi = t \).

Combining the labor supply (from part c), labor demand and profit (part g), gives:
\[
L = \alpha h - (1 - \alpha) \frac{\theta AK^\theta L^{1-\theta}}{(1 - \theta)AK^\theta L^{-\theta}}
\]
\[
L = \alpha h - (1 - \alpha) \frac{\theta}{1 - \theta} L
\]
\[
L \left[ 1 + (1 - \alpha) \frac{\theta}{1 - \theta} \right] = \alpha h
\]
\[
L \left[ \frac{1 - \alpha \theta}{1 - \theta} \right] = \alpha h
\]
\[
L^* = \frac{\alpha h (1 - \theta)}{1 - \alpha \theta}
\]

i. Suppose that initially $t_w = t_r = t = 30\%$, and equilibrium employment and equilibrium output were $L^* = 42$, $Y^* = 1200$. The government wants to stimulate the economy by lowering the tax rate to $t = 20\%$. Find the new equilibrium employment and equilibrium output. Explain your answer briefly.

$$L^* = 42, \quad Y^* = 1200$$

In a model with the same tax rate on both types of income, the government cannot affect the equilibrium employment or output by changing the common tax rate. The only effect of changes in taxes is on the division of output between private and government consumption.

j. In the last section, calculate the equilibrium private consumption and equilibrium government consumption ($C^*$, $G^*$), under $t = 30\%$ and $t = 20\%$.

$$t = 30\%$$

$$G^* = tY^* = 0.3 \cdot 1200 = 360$$

$$C^* = Y^* - G^* = 1200 - 360 = 840$$

$$t = 20\%$$

$$G^* = tY^* = 0.2 \cdot 1200 = 240$$

$$C^* = Y^* - G^* = 1200 - 240 = 960$$
2. (20 points). Consider the Keynesian model discussed in class. Suppose that the economy is characterized by the following behavioral functions:

Consumption: \[ C = C_0 + MPC(Y - T) \]

Investment: \[ I = I_0 \]

Government spending: \[ G = G_0 \]

Taxes: \[ T = T_0 + tY \]

In all the questions below, assume that \( Y^* < Y_f \).

a. Solve for the Keynesian equilibrium in the goods market.

\[
Y = E
\]

\[
Y = C_0 + MPC(Y - T_0 - tY) + I_0 + G_0
\]

\[
Y = MPC(1 - t)Y = C_0 - MPC \cdot T_0 + I_0 + G_0
\]

\[
Y^* = \frac{C_0 - MPC \cdot T_0 + I_0 + G_0}{1 - MPC(1 - t)}
\]

b. Suppose that the Keynesian multiplier is \( m_k = 4 \). Find the change in equilibrium output resulting from a decrease in investment by 10 units. In your answer you must show the formula used, before plugging the numbers.

\[
\Delta Y^* = m_k \cdot \Delta I_0 = 4 \cdot (-10) = -40
\]
c. Suppose that $Y^* = 880$, $Y_f = 1000$, the Keynesian multiplier is $m_k = 4$ and $MPC = 0.75$. What is the required change in the autonomous tax that would lead to equilibrium output at full employment? In your answer you must show the formula used, before plugging the numbers.

\[
\Delta Y^* = \frac{-MPC}{1 - MPC(1 - t)} \cdot \Delta T_0 \\
120 = -0.75 \cdot 4 \cdot \Delta T_0 \\
120 = -3 \cdot \Delta T \\
\Rightarrow \Delta T = -40
\]

d. On a fully labeled graph illustrate the Keynesian equilibrium in the goods market before and after the tax policy in the last section.

![Graph showing the Keynesian equilibrium](image-url)
3. (15 points). The following table contains data from the labor market of some country (in millions).

<table>
<thead>
<tr>
<th>Civilian noninstitutional population</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Civilian labor force</td>
<td>90</td>
</tr>
<tr>
<td>Employed</td>
<td>81</td>
</tr>
<tr>
<td>Unemployed</td>
<td>9</td>
</tr>
<tr>
<td>Not in the labor force</td>
<td>30</td>
</tr>
</tbody>
</table>

a. Complete the above table.

b. Find the unemployment rate in this country.

Unemp. Rate = \( \frac{\#\text{Unemp}}{\#\text{Labor Force}} = \frac{9}{90} = 0.1 = 10\% \)

c. Find the labor force participation rate in this country.

Labor Force Participation rate = \( \frac{\#\text{Labor Force}}{\#\text{Civilian Noninst. Pop.}} = \frac{90}{120} = \frac{3}{4} = 0.75 \)
4. (15 points). Consider the search model of unemployment, briefly described as follows.

<table>
<thead>
<tr>
<th>Fraction in population</th>
<th>Unemployed</th>
<th>Employed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U$</td>
<td>$1 - U$</td>
</tr>
<tr>
<td>Utility</td>
<td>$V_u(b, p, t_b)$</td>
<td>$V_e(w, s, t_w)$</td>
</tr>
</tbody>
</table>

$b$ – unemployment insurance benefit  
$p$ – probability of receiving a job offer  
$t_b$ – tax on $b$

$w$ – real wage  
$s$ – separation rate (probability of loosing the job)  
$t_w$ – tax on $w$

The symbols “+” under variable of the utility function indicates the assumption that the utility is increasing in that variable, and “−” under a variable indicates that the utility is decreasing in that variable.

**Distribution of wage offers:** $H(w)$ gives the probability that an offer is at least $w$.

a. Illustrate with 3 fully labeled graphs the impact of an increase in unemployment insurance benefits ($b \uparrow$) on: (1) reservation wage $w^*$, (2) probability of acceptance of job offers $H(w^*)$, and (3) steady-state unemployment rate $U^*$. 


b. Briefly discuss the intuition of the results you presented in the previous section.

Higher unemployment insurance benefits make the unemployed more comfortable staying unemployed, and more picky about accepting job offers. As a result, the reservation wage is higher and unemployed people accept fewer job offers (fewer job offers pay at least as much as the new reservation wage). The flow out of the unemployment is reduced, and unemployment rate rises.