LAB #5: FREQUENCY RESPONSE OF BJT AMPLIFIERS

(Updated Dec. 23, 2002)

Objective:
To investigate the frequency response of basic BJT amplifier configurations. To characterize a BJT dynamically. To compare measurements with computer simulations.

Components:
2 × 2N2222 *npn* BJTs, 1 × 1N4148 diode, 3 × 0.1μF capacitors, 1 × 10μF capacitor, 1 × 10 kΩ pot, and resistors: 1 × 10 Ω, 3 × 1.0 kΩ, 1 × 3.3 kΩ, 4 × 10 kΩ, and 1 × 1.0 MΩ (all 5%, ¼ W).

Instrumentation:
A dual adjustable regulated power supply, a digital multi-meter (DMM), a signal generator (sine wave, square wave), and a dual-trace oscilloscope.

PART I – THEORETICAL BACKGROUND

The dynamic characteristics of a BJT are controlled by *three* capacitive components:

- C_{je}, the *base-emitter*(B-E) *junction capacitance*
- C_{jc}, the *base-collector* (B-C) *junction capacitance*, usually denoted as C_μ in the analog literature
- C_b, the *base-charging capacitance*

For junction-isolated monolithic BJTs there is also a *fourth* capacitive component, namely, the *collector-to-substrate capacitance* C_{js}. Figure 1 shows the location of C_{je}, C_b, and C_μ in the BJT's small-signal model. For a BJT of the *npn* type, the expressions for the B-E and B-C junction capacitances are

$$C_{je} = \frac{C_{je0}}{\left(1 - v_{BE}/\phi_e\right)^{m_e}} \qquad\qquad C_\mu = \frac{C_{\mu 0}}{\left(1 - v_{BC}/\phi_c\right)^{m_c}} \qquad (1)$$

where:

- C_{je0} and $C_{\mu 0}$ are the *zero-bias values* of C_{je} and C_μ
- ϕ_e and ϕ_c are the *built-in potentials* of the B-E and the B-C junctions
- m_e and m_c are the *grading coefficients* of the B-E and B-C junctions
- v_{BE} and v_{BC} are the B-E and B-C *voltage drops*

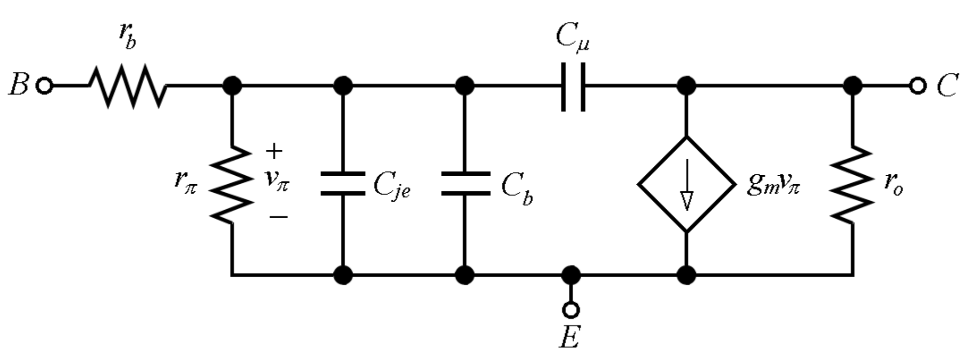


Fig. 1 – Small signal BJT model.

The base-charging capacitance C_b depends on the dc bias of the BJT as

$$C_b = \tau_F g_m \qquad \tau_F = \frac{W_B^2}{2D_b} \qquad g_m = \frac{I_C}{V_T} \tag{2}$$

where:

- τ_F is the *forward base transit time*
- g_m is the *transconductance*
- W_B is the *effective base-width*
- D_b is the *diffusion constant* of minority carriers in the base
- I_C is the *collector dc bias current*
- V_T is the *thermal voltage* (V_T = 26 mV at room temperature)

Since C_{je} and C_b are in parallel with each other, they are conveniently lumped together as a single equivalent capacitance C_π,

$$C_\pi = C_{je} + C_b \tag{3}$$

The maximum useful frequency of operation of a BJT is the frequency at which the small-signal current gain $\beta(jf)$ drops to *unity*. Aptly called the *transition frequency* f_T, it is expressed as

$$f_T = \frac{1}{2\pi}\frac{g_m}{C_\pi + C_\mu} \tag{4}$$

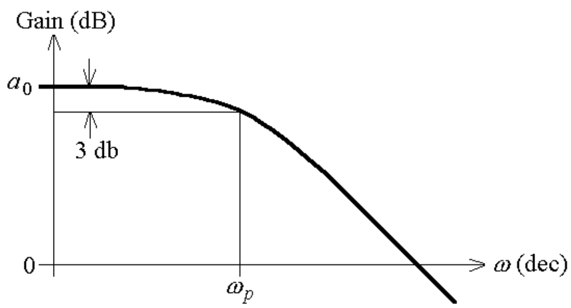
The above capacitances affect the dynamics of a BJT amplifier in different ways, depending on the particular configuration. As a rule, the number of *reactive elements* in an amplifier (capacitors in the present case) determines the number of *poles* in its gain $a(s)$, and thus the *order* of the system. Moreover, depending on circuit topology, gain may admit also *zeros*. For a physical system, the number n_z of zeros and the number n_p of poles are such that $n_z \le n_p$. Thus, with two net capacitances, namely, C_π and C_μ, a BJT is inherently a *2nd-order system*. In transistor circuits of practical interest the response is often dominated by just one pole at $s = -\omega_p$, indicating that gain can be approximated as

$$a(s) \cong \frac{a_0}{1 + s/\omega_p} \tag{5}$$

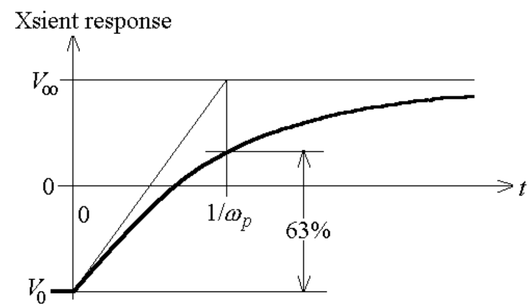
where a_0 is the low-frequency gain. The significance of the pole is two fold, depending on whether we are investigating the circuit's frequency response or transient response.

- In the *frequency domain*, depicted in Fig. 2*a*, ω_p represents the *–3-dB frequency* of the ac gain. This frequency is found experimentally by driving the circuit with a *sine wave* of constant-amplitude and variable-frequency, and then finding the frequency $\omega_{-3\text{ dB}}$ at which the output amplitude drops to $1/\sqrt{2}$, or 70.7% of its low-frequency value. Alternatively, it can be found as the frequency $\omega_{-45°}$ at which the output reaches a delay of –45° with respect to the input.

- In the *time domain*, depicted in Fig. 2*b*, $1/\omega_p$ represents the *time-constant* governing the exponential transient at the output. This constant is found experimentally by driving the circuit with a *square wave*, and then finding the time $t_{63\%}$ at which the output has completed $(1 - 1/e) \times (V_\infty - V_0)$, or 63% of the entire transition. Alternatively, it can be found as the time $t_{\text{intercept}}$ at which the *tangent* to the transient at V_0 (the initial value) intercepts V_∞ (the steady-state value).



(*a*)



(*b*)

Fig. 2 – Illustrating the meaning of ω_p in the (*a*) *frequency domain* and in the (*b*) *time domain*.

The Common-Emitter Amplifier

Of particular significance is the *common-emitter* (CE) configuration of Fig. 3*a*, which is conveniently analyzed using the *Miller approximation*. Such an approximation results in the simplified ac equivalent of Fig. 3*b*, where

$$C_t = C_\pi + C_M \tag{6}$$

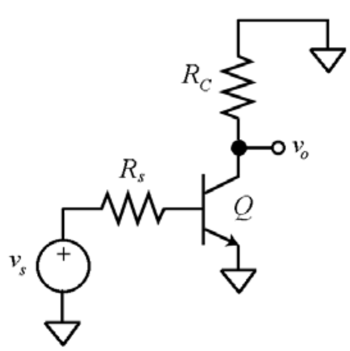
is called the *total equivalent capacitance* between base and emitter, and

$$C_M = (1 + g_m R_o)C_\mu \tag{7}$$

is the *Miller capacitance*. In the above expression,

$$R_o = R_C // r_o \tag{8}$$

is the amplifier's *output resistance.* Here, $r_o = V_A/I_C$, where V_A is the *Early voltage* of the BJT. The *small-signal voltage gain* of the CE configuration is readily found to be



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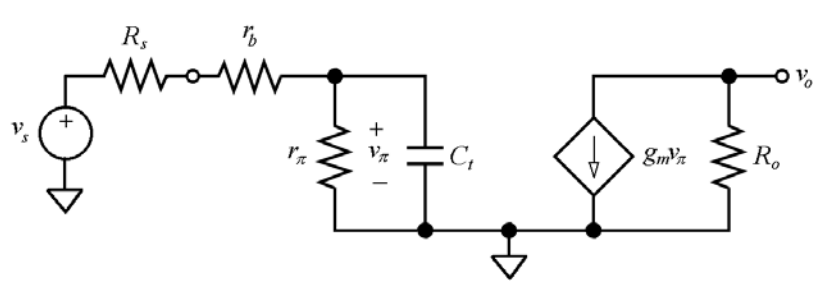


(*b*)

Fig. 3– (*a*) *Common-emitter* (CE) ac equivalent, and (*b*) its small-signal model using the *Miller approximation.*

$$a(jf) \cong \frac{a_0}{1 + jf/f_b} \tag{9}$$

where a_0 is the *low-frequency gain* and f_b is the *–3-dB frequency*,

$$a_0 = \frac{r_\pi}{R_s + r_b + r_\pi}(-g_m R_o) \tag{10a}$$

$$f_b = \frac{1}{2\pi R_t C_t} \tag{10b}$$

In the above expressions, $r_\pi = \beta_0/g_m = \beta_0 V_T/I_C$, where β_0 is the low-frequency small-signal current gain; r_b is the *bulk resistance* of the base region; R_t is the *equivalent resistance* seen by C_t, or

$$R_t = (R_s + r_b)//r_\pi \tag{11}$$

The *gain-bandwidth product* is defined as $GBP = |a_0 \times f_b|$. For the CE amplifier, its expression is

$$GBP = |a_0 \times f_b| = \frac{g_m R_o}{2\pi(R_s + r_b)C_t} \tag{12}$$

PART II – EXPERIMENTAL PART

In this lab we are going to investigate the frequency responses of the basic BJT configurations using the 2N2222 discrete *npn* BJT. The advantage of working with this particular BJT is that its model is already available in PSpice's Library, so you can always run PSpice simulations to anticipate what to expect in the lab. Also, you are encouraged to compare the findings for your *particular* 2N2222 sample with the *typical* data tabulated in the data sheets. The latter can be downloaded from the Web (for instance, by visiting http://www.google.com and searching for "2N2222" or variants thereof.)

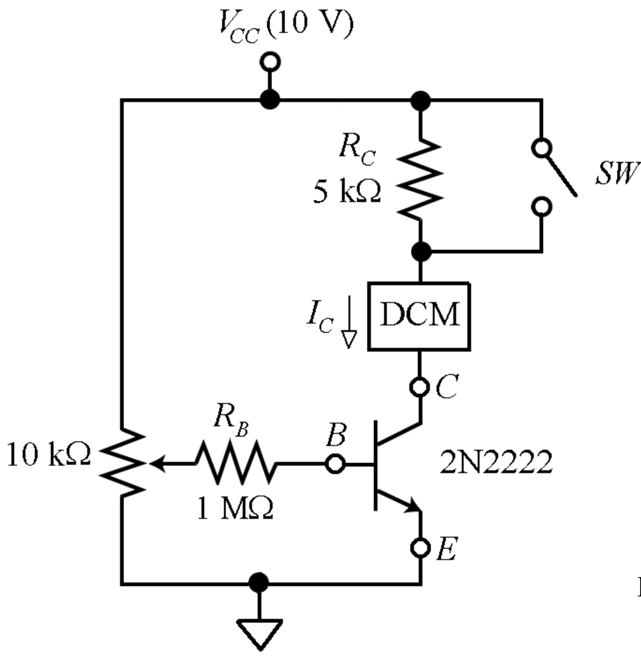
As you assemble your circuits, keep leads short, bypass all power supplies with 0.1-μF capacitors mounted closely to the circuit under investigation, and don't forget to turn power off whenever you make any circuit changes. Also, to reduce dynamic loading of your circuit by the oscilloscope, use *low input-capacitance* probes, such as X10-probes.

Henceforth, steps shall be identified by letters as follows: **C** for calculations, **M** for measurements, and **S** for SPICE simulation.

Preliminary BJT Characterization

We begin with some preliminary BJT characterizations, whose results will be needed subsequently. Thus, mark one of the 2N2222 BJTs in your kit, and proceed as follows.

MC1: With power off, assemble the circuit of Fig. 4 (implement the 5-kΩ resistor using 2 × 10 kΩ resistors in parallel). Keep leads short, and bypass the power-supply to ground via a 0.1-μF capacitor, as recommended in the Appendix. Then, apply power, and while monitoring I_C with the *digital current meter* (DCM), adjust the potentiometer until I_C = 1.0 mA. Next, short out R_C with a wire (that is, close *SW*) so as to effect the change ΔV_{CE} = 5 V and record the corresponding change ΔI_C (this change will be small, so use as many digits as your instrument will allow). Finally, compute $r_o = \Delta V_{CE}/\Delta I_C$.



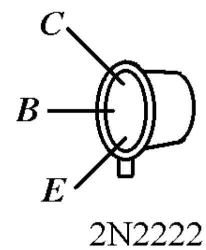


Fig. 4 - Test circuit to find r_o.

MC2: With power off, assemble the circuit of Fig. 5, keeping leads short and bypassing both supplies to ground via two 0.1-μF capacitors. Apply power, and while monitoring I_C with the DCM inserted in *series* at node C, adjust the dual tracking supplies until $I_C = 1.0$ mA. Next, shut off power, insert the DCM in series at node B, and measure I_B. Finally, calculate $\beta = I_C/I_B$, $g_m = I_C/V_T$, and $r_\pi = \beta/g_m$.

The Common Emitter (CE) Configuration

Next, we turn to CE amplifier of Fig. 6. Here, we use the voltage divider made up of R_1 and R_2 for the dual purpose of scaling down the signal generator v_s to a signal $v_i = v_sR_2/(R_1 + R_2) = v_s/99$ that meets the small-signal requirements of the CE amplifier, and to ensure a low and predictable source resistance

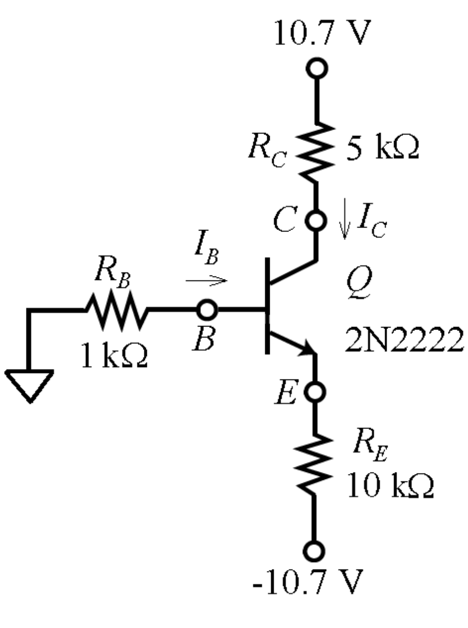


Fig. 5 - Test circuit to measure I_C and I_B, and thus find β, g_m, and r_π.

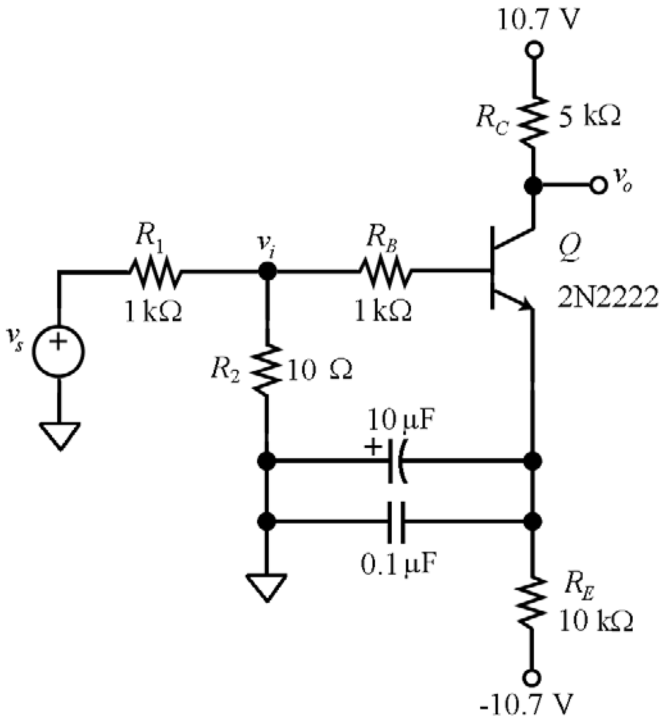


Fig. 6 – The CE amplifier.

$R_S = R_1//R_2 \cong 10\ \Omega$. Moreover, to ensure a good-quality ac ground at the emitter over a broad range of frequencies, we use the parallel combination of a 0.1-µF disc capacitor and a 10-µF polarized capacitor. Polarized capacitors tend to exhibit significant inductive behavior at high frequencies, so it is always good practice to parallel a polarized type with a disc type to achieve a composite device capable of ensuring a good ac ground over a wide range of frequencies.

We observe that our amplifier will exhibit a *high-frequency pole* at f_b as predicted by Eq. (10*b*), but also a *low-frequency pole* at $f_a = 1/(2\pi R_e C_e)$, where R_e is the equivalent resistance seen looking into the emitter, and C_e is the net emitter capacitance ($C_e = 10 + 0.1 \cong 10$ µF). It is thus understood that in the present context what is referred to as the *low-frequency gain* a_0 of Eq. (10*a*) denotes the gain over the frequency range $f_a << f << f_b$. For obvious reasons, this gain is also called the *mid-band gain*, and f_a and f_b the *lower and upper pole-frequencies*.

MC3: With power off, assemble the circuit of Fig. 6, keeping leads short and with the ground side of the capacitors connected *directly* to the ground side of R_2, as shown. Also, be sure that the "+" plate of the polarized capacitor goes to ground, and the "–" plate to the emitter, which is at a negative voltage. Next, adjust the waveform generator so that v_s is a 100-kHz *sine wave* with 0-V DC offset and 1-V *peak-to-peak* amplitude (this makes the amplitude of v_i about 10-mV peak-to-peak, small enough to guarantee the validity of the small-signal model). To reduce dynamic loading of your circuit by the oscilloscope, use *low input-capacitance* probes, such as X10-probes. Then proceed as follows:

- Apply power, and while monitoring v_s and v_o with Ch. 1 and Ch. 2 of the oscilloscope, both channels set on AC, vary the generator's frequency up and down the frequency spectrum to observe that gain is fairly *constant* at mid frequencies, but *rolls off* both at the *low* and *high* ends of the spectrum.
- Measure the *mid-band gain* from v_s to v_o, and multiply it by 99 to obtain the amplifier's mid-band gain a_0 from v_i to v_o.
- Lower the frequency until v_o drops to 70.7% of its mid-band value. This is the aforementioned

lower pole frequency f_a. Record its experimental value, and compare with its theoretical value $f_a = 1/(2\pi R_e C_e)$. Are they close enough? Account for any possible differences.

- Raise the frequency until v_o again drops to 70.7% of its mid-band value. This is the *upper pole frequency* f_b. Record its experimental value, and compare with its theoretical value predicted by Eq. (10*b*). Are they close enough? Account for any possible differences.
- Using the experimental values of a_0 and f_b, calculate $GBP = |a_0 \times f_b|$.

S4: Simulate the circuit of Fig. 6 via PSpice using the 2N2222 model available in PSpice's Library. Hence, use the cursor facility of PSpice to determine a_0, f_a, and f_b. Considering that your measurements are based on a *particular* 2N2222 sample, while the PSpice model is based on *typical* 2N2222 data, comment on the degree of agreement between measurements and simulation.

MC5: With power off, set $R_B = 0$ in the circuit of Fig. 6 by replacing R_B with a plain wire. Reapply power, and retrace the procedure of Step MC3 to find the new values of a_0, f_b, and *GBP*. Compare with those of Step MC3, and comment on your findings.

Note: With $R_B = 0$, a_0 and f_b will increase. Should f_b exceed the upper frequency limit of your waveform generator, you can estimate it via *time-domain techniques*, as illustrated in Fig. 2*b*. That is, you first measure the time constant τ governing the transient response, and then calculate $f_b = 1/(2\pi\tau)$.

S6: Simulate the circuit of Step MC5 via PSpice, and comment on the degree of agreement between measurements and simulation.

C7: Steps MC3 and MC5 are designed to exploit Eq. (12) to establish two equations in the two unknowns r_b and C_t (the remaining parameters can be calculated directly), namely,

$$GBP_1 = \frac{g_m R_o}{2\pi(R_s + R_B + r_b)C_t} \qquad (13a)$$

$$GBP_2 = \frac{g_m R_o}{2\pi(R_s + r_b)C_t} \qquad (13b)$$

where GBP_1 is the gain-bandwidth product obtained in Step MC3 with $R_B = 1$ kΩ, and GBP_2 is that obtained in Step MC5 with $R_B = 0$. Moreover, $R_S \cong 10$ Ω, and the product $g_m R_o$ is readily calculated. Thus, solve Eqs.(13) to estimate r_b and C_t. Are their values typical?

Hint: First, take the ratio GBP_2/GBP_1, and solve for r_b. Then, back substitute into Eq. (13*b*) and solve for C_t.

MC8: With power off, assemble the circuit of Fig. 7 (implement the 11-kΩ resistor as 10 kΩ in series with 1.0 kΩ.) Then , turn power on, and measure the new value of f_b.

Note that this circuit is the same as that of Fig. 6, except for the modified resistive network seen by the collector, which is designed to *change the value* of R_o while leaving everything else the same, including the bias conditions of the collector and thus the value of C_μ. Consequently, we now have two equations in the two unknowns C_π and C_μ (the remaining parameters can be calculated directly), namely,

$$f_{b1} = \frac{1}{2\pi R_t[C_\pi + (1 + g_m R_{o1})C_\mu]} \qquad (14a)$$

$$f_{b2} = \frac{1}{2\pi R_t[C_\pi + (1 + g_m R_{o2})C_\mu]} \qquad (14b)$$

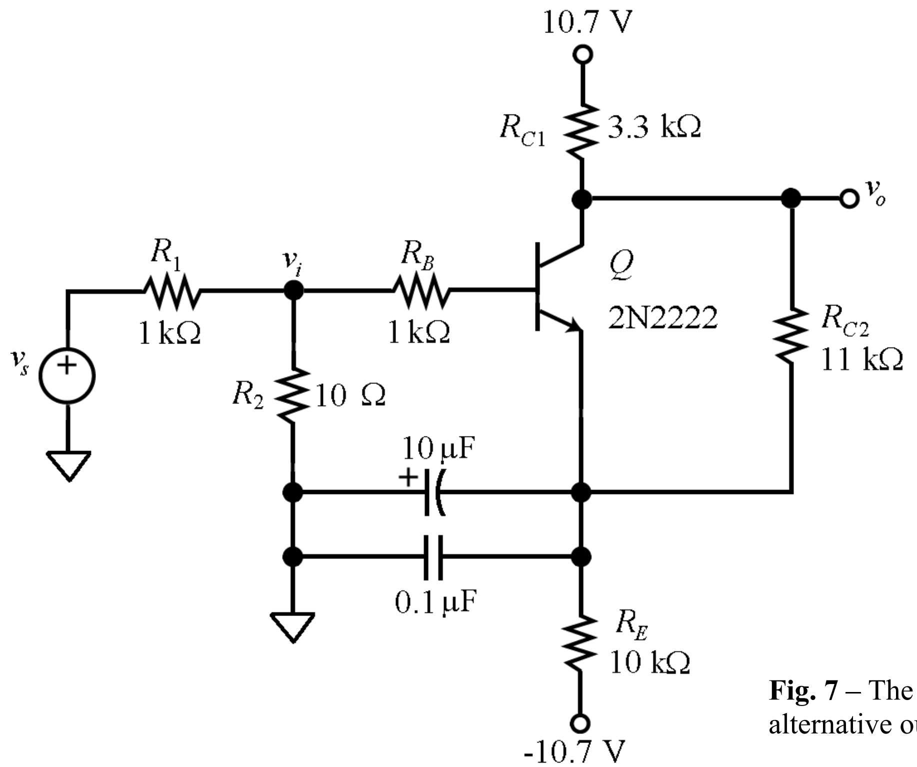


Fig. 7 – The CE amplifier with an alternative output-resistance setting.

Here, f_{b1} is the experimental –3-dB frequency of the circuit of Fig. 6, whose output resistance is calculated as $R_{o1} = (10\ \text{k}\Omega)//r_o$. Likewise, f_{b2} is the experimental –3-dB frequency of the circuit of Fig. 7, whose output resistance is calculated as $R_{o2} = (3.3\ \text{k}\Omega)//(11\ \text{k}\Omega)//r_o$. Moreover, R_t is calculated in both cases via Eq. (11). Thus, solve Eqs. (14) to estimate C_π and C_μ for this particular BJT sample and under the present collector bias conditions. Are their values typical?

Hint: First, take the difference $1/f_{b2} - 1/f_{b1}$, and solve for C_μ. Then, back substitute into Eq. (14*a*) and solve for C_π.

The Cascode Configuration:

In the common-base (CB) configuration, one side of C_μ is at ac ground, so this configuration does not suffer from the Miller effect and is inherently faster than the CE configuration. However, it exhibits much lower input impedance than the CE configuration, and this is generally a drawback in voltage-type amplification. The *cascode* configuration ingeniously combines the advantages of both the CE and the CB configurations, if at the expense of an additional BJT. With reference to Fig. 8, we note that the low-frequency gain of the CE stage is only about –1 V/V, indicating that the Miller effect increase C_μ only by a factor of 2, much less than in the basic CE amplifier of Fig. 6.

The mid-frequency gain of the cascode configuration can still be found via (10*a*), but with $R_o \cong R_C$. Moreover, over the frequency range of interest, the response is governed by the dominant pole of Q_1, indicating that the upper pole frequency can be estimated as

$$f_b \cong \frac{1}{2\pi[(R_s + r_b)//r_\pi]\times[C_\pi + 2C_\mu]} \tag{16}$$

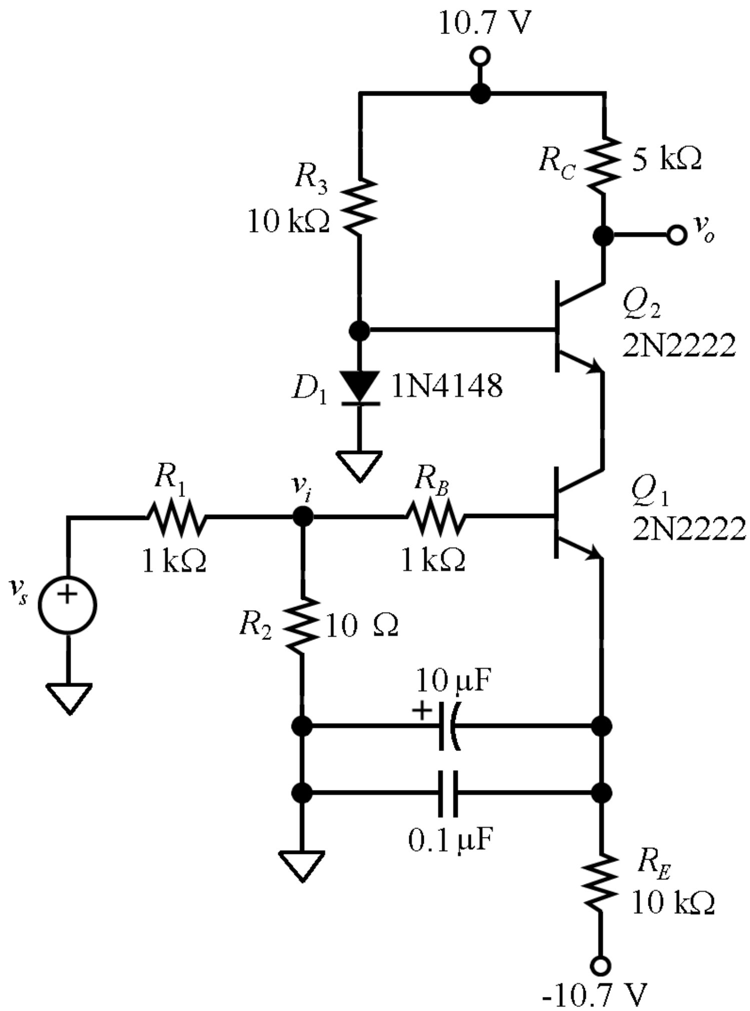


Fig. 8 – The CE-CB, or *cascode* amplifier.

9: With power off, assemble the circuit of Fig. 8. Note that the circuit is similar to that of Fig. 6, except for the insertion of Q_2 in series between Q_1 and R_C, and the addition of R_3 and D_1 to suitably bias Q_2. Then, apply power and proceed along the lines of Step MC3 to measure the *mid-band gain* a_0 as well as the upper pole-frequency f_b. What is the *GBP* of your amplifier? How does it compare with that of the CE amplifier?

Note: Should f_b exceed the upper frequency limit of your waveform generator, estimate it via *time-domain techniques*, as illustrated in Fig. 2*b*. That is, you first measure the time constant τ governing the transient response, and then calculate $f_b = 1/(2\pi\tau)$.

S10: Simulate the circuit of Fig. 8 via PSpice using the 2N2222 and 1N4148 models available in PSpice's Library. Hence, use the cursor facility of PSpice to determine a_0 and f_b. Considering that your measurements are based on two *particular* 2N2222 samples, while the PSpice models are based on *typical* 2N2222 data, comment on the degree of agreement between measurements and simulation.

The CC Configuration:

Like the CB amplifier, the common-collector (CC) amplifier, also called *emitter follower,* does not suffer from the Miller effect; so it too is an inherently fast configuration. Moreover, as shown in the version of Fig. 9, it lends itself to be driven directly from the input source with no need for any external capacitors,

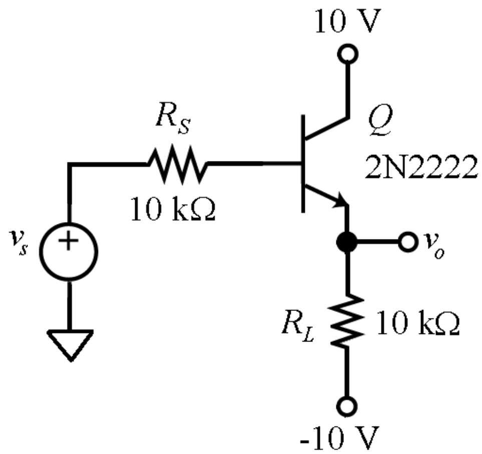


Fig. 9 – The common-collector (CC) amplifier.

indicating its ability to operate all the way down to DC.

M11: With power off, assemble the circuit of Fig. 9, and adjust the waveform generator for a 5-V peak-to-peak, 0-V DC offset *sine wave*. Then, apply power, and increase the input frequency until the output amplitude drops to 70.7% of its low-frequency value. Clearly, this represents the -3-dB frequency of your circuit.

Note: Should $f_{-3\text{ dB}}$ exceed the upper frequency limit of your waveform generator, estimate it via *time-domain techniques*, as illustrated in Fig. 2*b*. That is, you first measure the time constant τ governing the transient response, and then calculate $f_b = 1/(2\pi\tau)$.

S12: Simulate the emitter follower of Fig. 9 via PSpice using the 2N2222 model available in PSpice's Library. Display the *magnitude plot* of its *gain* $a(jf) = V_o/V_s$, and use the cursor facility to find its *dc value* as well as its *–3-dB frequency*.

S13: Using PSpice, along with a suitable test source, first at the input, then at the output, display the magnitude plots of the *input impedance* $z_i(jf)$ seen looking *into the base* and the output impedance $z_o(jf)$ seen looking *into the emitter* of the CC amplifier of Fig. 9 (use log-log scales). Hence, use the cursor facility to find their low-frequency and high-frequency *asymptotic values* as well as their *pole* and *zero* frequencies Comment on your results. And be a happy engineer!