

## Modern Growth Theory

Robert Solow won the Nobel prize in economics (in 1987) for his contribution to the Economic theory of Growth. The Solow model is superior to the Harrod-Domar (HD) model. The HD model follows a Leontief production function which implies: (1) labor and capital are used in fixed proportions; (2) the ration of output to capital is also fixed; (3) no diminishing returns. All of these assumptions are unrealistic.

The model presented below is similar to the model created by Solow, but contains one significant modification. In our model, technology is assumed to be output-augmenting (Hicks neutral.) In the Solow model, technology is assumed to be labor augmenting (Harrod-neutral.) We will discuss labor augmenting technologies later in the semester.

### I. Growth Theory without Technological Change

#### Definitions

- A.  $Y$  = Total output (GDP)       $y$  = income per capita
- B.  $S$  = Total Savings       $s$  = savings rate ( $0 < s < 1$ )
- C.  $L$  = total number of workers (labor)
- D.  $K$  = Capital       $k$  = capital per worker =  $K/L$
- E.  $n$  = the growth rate of labor ( $0 < n < 1$ )
- F.  $d$  = rate of depreciation of capital ( $0 < d < 1$ )
- G.  $f(K, L)$  is a production function converting capital and labor into output;  $f$  exhibits diminishing returns to scale

#### Assumptions

- A. Constant returns to scale
- B.  $y = f(k)$   $f$  also exhibits diminishing returns
- C.  $s = \mathbf{s}$        $s$  is constant
- D.  $n = \mathbf{n}$        $n$  is constant
- E. As in the Harrod-Domar model, we assume a closed economy and in equilibrium  $S = I$ .

For each individual total savings are equal to  $sf(k)$  and investment is just the change in the capital stock, so:

$$\Delta k = sf(k)$$

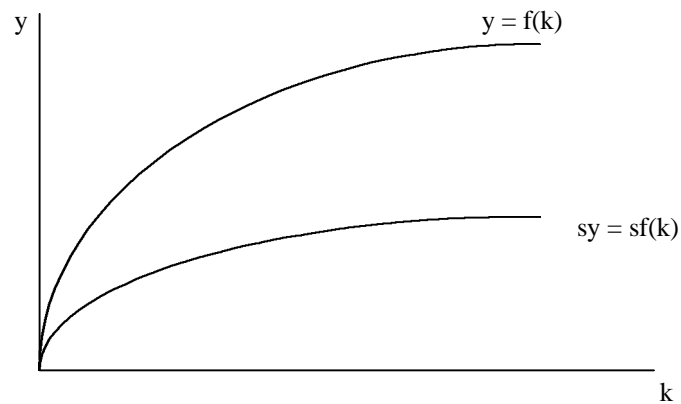
However, since the population is growing at a rate of  $n$  and capital is depreciating at rate  $d$ :

$$\Delta k = sf(k) - dk - nk$$

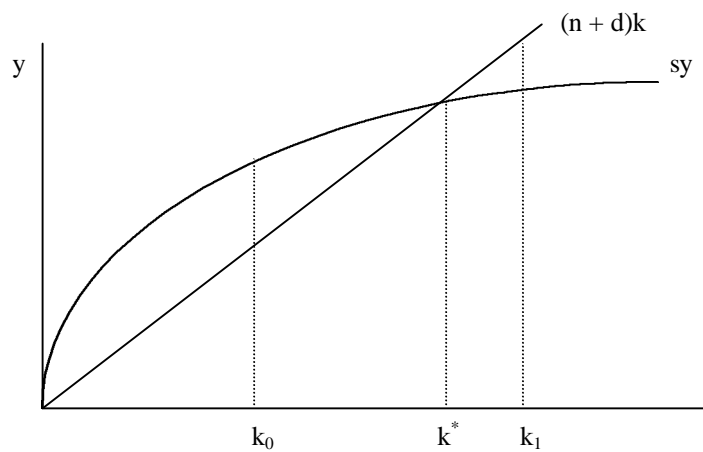
OR

$$\Delta k = sy - (d + n)k$$

We can illustrate this graphically as follows:

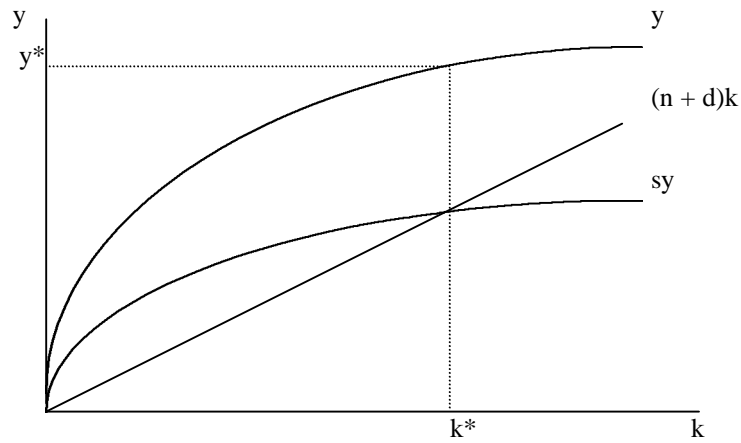


We are concerned with the equilibrium or steady state:



In the above diagram, at  $k_0$  gross investment,  $sy$ , exceeds the outflow of capital per worker from depreciation and population growth  $(n + d)k$ . Therefore the capital stock will grow  $[\Delta k > 0]$ . Capital will continue to appreciate until  $k^*$  is reached. Similarly, if the economy is at  $k_1$  then outflow exceeds inflow and the capital stock will fall until it reaches  $k^*$   $[\Delta k < 0]$ . So, the economy will always move towards the point  $k^*$ . This point is called the **steady state** or equilibrium. **At this point  $\Delta k = 0$  or  $sy = (n+d)k$**

Combining the above two graphs:

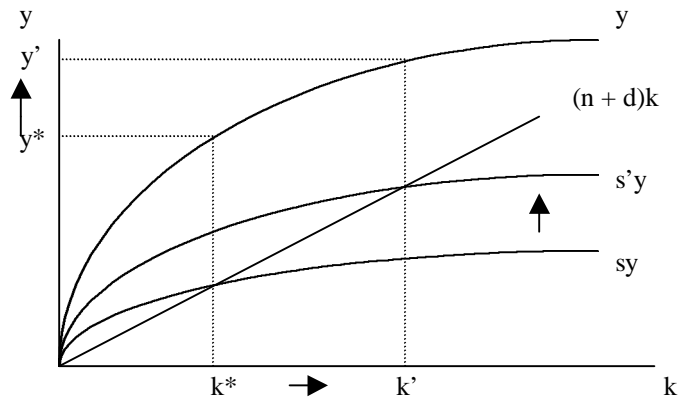


In the above diagram,  $y^*$  is steady state output per worker.

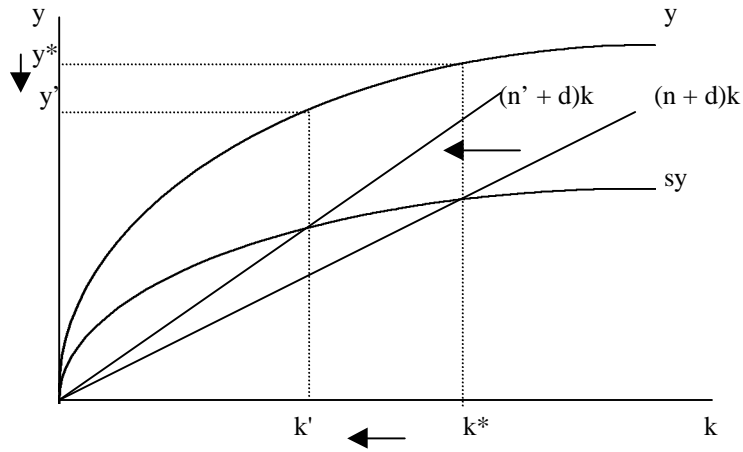
What is the growth rate of income per capita in the above diagram? (A: zero)

Is it possible to explain economic growth per capita with the above theory? Yes. Growth may occur: (1) if  $s$  increases; (2) if the rate of population growth falls.

**Example:** Suppose the rate of savings increases from  $s$  to  $s'$ . This increase in savings increases capital per worker, which is often referred to as **capital deepening**. Capital deepening is one explanation for the growth of the Asian Tigers



Now, suppose that the population rate grows from  $n$  to  $n'$ :



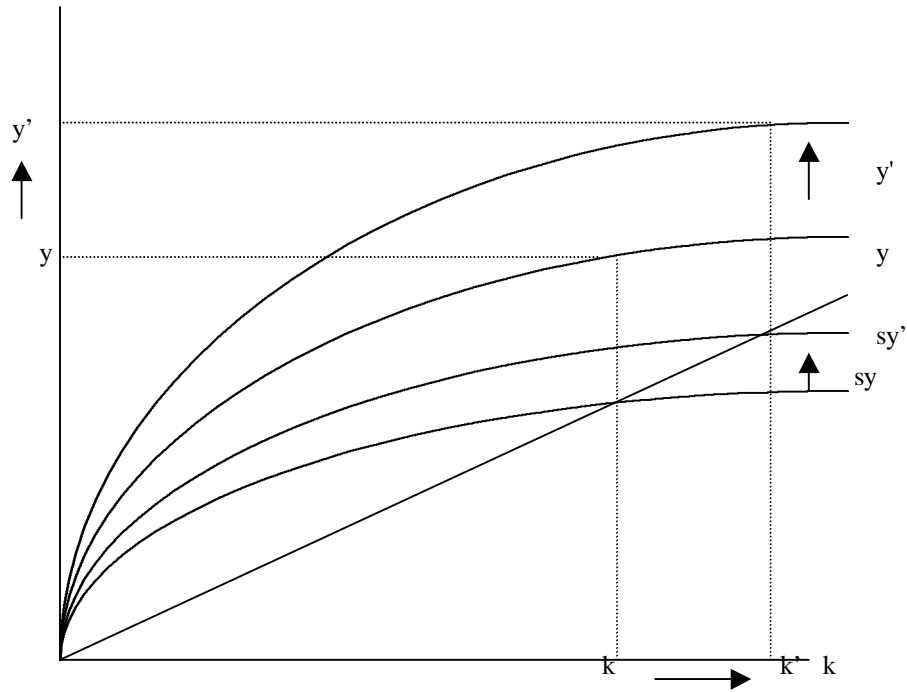
The problem with the above model is that there is no explanation for growth in output per worker other than an increase in savings or decreases in population. Obviously something is wrong. The main problem is that we have not yet incorporated technology into the model.

## II The Model with Hicks neutral Technology

Additional Assumptions:

- A. Technology is defined by the variable A
- B.  $y = Af(k)$
- C. the rate of growth of technology is given by  $g$  ( $0 < g < 1$ )

Now, let us look at the effect of an increase in Technology from A to A'. So, y increases from y to y'.



## III Endogenous growth models and the “New” growth theory

More recent models of economic growth attempt to explain why and how technology changes. These models make technology part of the model (endogenous) as opposed to the model presented in section II, which assumes that technology is determined outside the model (exogenous.)