Problem 1. (20pts) Evaluate the line integral \( \int_C xy \, dx + (x + y) \, dy \) along the parabola \( y = x^2 \) from \((-1, 1)\) to \((2, 4)\).
Problem 2. (20pts) Determine whether or not the vector field \( \mathbf{F} = (y + z)\mathbf{i} + (x + z)\mathbf{j} + (x + y)\mathbf{k} \) is conservative, and if so, find a potential function for it.
Problem 3. (20pts) Find the counter-clockwise circulation of \( \mathbf{F} = (x - y)\mathbf{i} + (y - x)\mathbf{j} \) along the curve \( C \) around the square bounded by \( x = 0, x = 1, y = 0, y = 1 \).
Problem 4. (20pts) Find the surface area of the lower portion cut from the sphere \( x^2 + y^2 + z^2 = 2 \) by the cone \( z = \sqrt{x^2 + y^2} \).
Problem 5. (20pts) Find the outward flux of $\mathbf{F} = (y - x)\mathbf{i} + (z - y)\mathbf{j} + (y - x)\mathbf{k}$ along the boundary of the cube $E$ bounded by the planes $x = \pm 1$, $y = \pm 1$, $z = \pm 1$. 
Problem 6. (20pts) Find the outward flux of $\mathbf{F} = x^2 \mathbf{i} + y^2 \mathbf{j} + z^2 \mathbf{k}$ across the boundary of the region $D$ cut from the solid cylinder $x^2 + y^2 \leq 4$ by the planes $z = 0$ and $z = 1$. 
Problem 7. (20pts) A space probe in the shape of the ellipsoid

\[ 4x^2 + y^2 + 4z^2 = 16 \]

enters Earth’s atmosphere and its surface begins to heat. After 1 hour, the temperature at point \((x, y, z)\) on the probe’s surface is

\[ T(x, y, z) = 8x^2 + 4yz - 16z + 600. \]

Find the hottest point on the probe’s surface.
Problem 8. (30pts) Short Answers.

1. For what values of $\alpha$ is $r^\alpha \hat{r}$ conservative? (Here, $r = \sqrt{x^2 + y^2 + z^2}$.)

2. TRUE/FALSE: Every conservative vector field is irrotational.

3. TRUE/FALSE: Every irrotational vector field defined on all of 3-space is conservative.

4. Give an example of an irrotational vector field that is not conservative. Be sure to specify the domain.

5. Which of the following conditions guarantees that $\int_C \mathbf{F} \cdot d\mathbf{r}$ depends only on the endpoints of the path $C$.
   (A) $\nabla \cdot \mathbf{F} = 0$
   (B) $\nabla \times \mathbf{F} = 0$
   (C) $\mathbf{F} = \nabla f$ for some scalar-valued function $f$
   (D) $\mathbf{F} = \nabla \times \mathbf{G}$ for some vector field $\mathbf{G}$

6. Which of the following conditions guarantees that $\int_S \mathbf{F} \cdot d\mathbf{r}$ depends only on the values of $\mathbf{F}$ on the boundary of $S$.
   (A) $\nabla \cdot \mathbf{F} = 0$
   (B) $\nabla \times \mathbf{F} = 0$
   (C) $\mathbf{F} = \nabla f$ for some scalar-valued function $f$
   (D) $\mathbf{F} = \nabla \times \mathbf{G}$ for some vector field $\mathbf{G}$
Problem 9. (30pts) Stokes Theorem

(a) Let
\[ \hat{\mathbf{r}} = \frac{x \mathbf{i}}{\sqrt{x^2 + y^2}} + \frac{y \mathbf{j}}{\sqrt{x^2 + y^2}} \text{ and } \hat{\mathbf{\theta}} = \frac{-y \mathbf{i}}{\sqrt{x^2 + y^2}} + \frac{x \mathbf{j}}{\sqrt{x^2 + y^2}}. \]

Let \( C \) be the circle of radius 3 centered at the origin. Compute the flux of the vector fields \( \hat{\mathbf{r}} \) and \( \hat{\mathbf{\theta}} \) across \( C \) with respect to the outward normal?

(b) IDENTIFY the true statement(s) among the following.

(A) \[ \iint_{\partial E} \mathbf{F} \cdot d\mathbf{\sigma} = \iiint_{E} (\nabla \cdot \mathbf{F}) \, dV \]

(B) \[ \iint_{\partial E} (\nabla \cdot \mathbf{F}) \, d\mathbf{\sigma} = \oint_{\partial S} \mathbf{F} \cdot d\mathbf{r} \]

(C) \[ \iiint_{E} (\nabla \cdot \mathbf{F}) \, dV = \iint_{S} |\mathbf{F}| \, dV \]

(D) \[ \oint_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \iiint_{S} (\nabla \times \mathbf{F}) \cdot d\mathbf{\sigma} \]

(E) \[ \iiint_{S} \mathbf{F} \cdot d\mathbf{\sigma} = \oint_{\partial S} (\nabla \times \mathbf{F}) \cdot d\mathbf{r} \]

(F) \[ \iint_{\partial E} (\nabla \mathbf{F}) \, d\mathbf{\sigma} = \iiint_{E} \mathbf{F} \, dV \]