Math 227: Calculus III

Midterm III

Fall 2006

Name: KEY

NOTE: There are 5 problems on this midterm (total of 6 pages). Use of calculators to check your work is permitted; however, in order to receive full credit for any problem, you must show work leading to your answer. You have 45 minutes to complete this test.

Date: December 7, 2006

#students = 31

Median = 79

Mean = 76.5 ± 13.2

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<th>Problem</th>
<th>Possible points</th>
<th>Score</th>
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<55 | 55+ | 60+ | 65+ | 70+ | 75+ | 80+ | 85+ | 90+ | 95+ |

0 3 4 4 3 3 4 2 6 2
Problem 1. (20pts) Determine whether the following series converges or diverges. Then, if the series converges, determine whether it converges conditionally or absolutely. Be sure to carefully explain how you reached your conclusions by describing the kind of test your are using, why the test you are using applies, and how you used the test to reach your conclusion.

\[ \sum_{n=1}^{\infty} \left( \frac{3}{5^n} + \frac{2}{n} \right) \]

\[ = \sum_{n=1}^{\infty} \frac{3}{5^n} + \sum_{n=1}^{\infty} \frac{2}{n} \]

\[ \sum_{n=1}^{8} \frac{3}{5^n} \] converges \( \text{bac geometric, } \text{ratio } = \frac{1}{5} < 1 \)

\[ \sum_{n=1}^{\infty} \frac{2}{n} \] diverges \( \text{bac harmonic (p-series, p = 1)} \)

\[ \sum_{n=1}^{\infty} \left( \frac{3}{5^n} + \frac{2}{n} \right) \] also diverges.
Problem 2. (20pts) Determine whether the following series converges or diverges. Then, if the series converges, determine whether it converges conditionally or absolutely. Be sure to carefully explain how you reached your conclusions by describing the kind of test your are using, why the test you are using applies, and how you used the test to reach your conclusion.

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n + n^2} \]

\[ \sum \frac{1}{n+n^2} \leq \sum \frac{1}{n^2} \quad \text{converges (p-series, p>1)} \]

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n+n^2} \quad \text{is absolutely convergent.} \]

(Note: Any absolutely convergent series is convergent.)
Problem 3. (20pts) Determine whether the following series converges or diverges. Then, if the series converges, determine whether it converges conditionally or absolutely. Be sure to carefully explain how you reached your conclusions by describing the kind of test your are using, why the test you are using applies, and how you used the test to reach your conclusion.

\[ \sum_{n=1}^{\infty} \frac{1}{n^2 - 4n + 5} \]

limit comparison with \( \sum \frac{1}{n^2} \).

\[ \lim_{n \to \infty} \frac{n^2}{n^2 - 4n + 5} = \lim_{n \to \infty} \frac{1}{1 - \frac{4}{n} + \frac{5}{n^2}} = 1 > 0 \]

\[ \sum_{n=1}^{\infty} \frac{1}{n^2 - 4n + 5} \] converges since \( \sum \frac{1}{n^2} \) does

\((p\text{-series}, p>1)\)
Problem 4. (20 pts) Determine whether the following series converges or diverges. Then, if the series converges, determine whether it converges conditionally or absolutely. Be sure to carefully explain how you reached your conclusions by describing the kind of test your are using, why the test you are using applies, and how you used the test to reach your conclusion.

\[ \sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n} \]

This is an alternating series with terms

[\[ |a_n| = \frac{\ln n}{n} \quad \text{eventually decreasing to 0.} \]

Indeed, \( f(x) = \frac{\ln x}{x} \) has

[\[ f'(x) = \frac{1 - \ln x}{x^2} < 0 \quad \text{for } x > e. \]

\[ \therefore \text{ since } \lim_{n \to \infty} a_n = 0 \text{ alternating series test implies } \sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n} \text{ converges.} \]

Since \[ \frac{\ln n}{n} > \frac{1}{n} \quad \text{for } n > e, \] direct comparison with harmonic series shows

\[ \sum_{n=1}^{\infty} \frac{\ln n}{n} \text{ diverges.} \]

\[ \therefore \sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n} \text{ is conditionally convergent.} \]
**Problem 5.** (20pts) Determine whether the following series converges or diverges. Then, if the series converges, determine whether it converges conditionally or absolutely. Be sure to carefully explain how you reached your conclusions by describing the kind of test your are using, why the test you are using applies, and how you used the test to reach your conclusion.

\[ \sum_{n=1}^{\infty} \frac{2^n}{n^4} \]

\[ \lim_{n \to \infty} \frac{2^n}{n^4} = \infty \neq 0 \]

**\( \infty \) divergence test** \( \Rightarrow \) \( \sum_{n=1}^{\infty} \frac{2^n}{n^4} \) diverges.

Alternatively, ratio test could be used:

\[ \lim_{n \to \infty} \frac{2^{n+1}}{(n+1)^4} \cdot \frac{n^4}{2^n} = \lim_{n \to \infty} \frac{2}{(1+\frac{1}{n})^4} = 2 > 1 \]