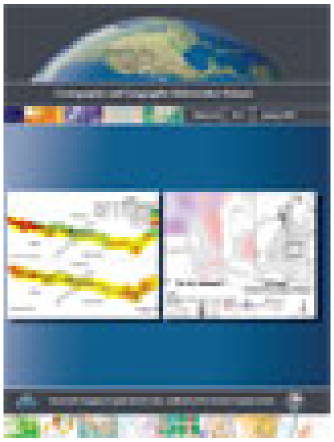

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The “M” in digital elevation models

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The "M" in digital elevation models

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The "M" in digital elevation models (DEM) stands for model, which literally means "a schematic description of a system, theory, or phenomenon that accounts for its known or inferred properties and may be used for further study of its characteristics." A DEM fulfills the requirement of "a schematic description" of terrain. However, how to make it account for the "known or inferred properties" warrants further scrutiny. This article outlines three properties of terrain and examines their four implications to DEM generation. The three properties are as follows: (1) each terrain point has a single, fixed elevation; (2) terrain points have an order and sequence that is determined by their elevations; and (3) terrain has skeletons. The four implications to DEM generation methods are as follows: (1) a method must be a bijection; (2) a method must be an isomorphism in order to preserve elevation sequence; (3) a method must guarantee that the vertical error at *any* point, not just checkpoints, is acceptable in order to assure the vertical accuracy of a DEM; and (4) a method must involve generalization if terrain skeletons are to be preserved. These implications are discussed in the context of light detection and ranging-derived DEMs. Generalization is highlighted as the top priority for future research.

Keywords: DEM; terrain modeling; generalization; isomorphism

Introduction

Digital elevation models (DEM) provide digital topography and bathymetric data that are essential for a myriad of applications such as telecommunication, hydrologic modeling, military defense, and disaster preparedness. Although the term "DEM" is widely used, its exact meaning warrants further scrutiny. Generally speaking, a DEM is a digital representation of terrain or the surface topography of the earth (Longley et al. 2005). However, each letter in DEM has multiple interpretations. The "D" in DEM stands for digital, which, while self-explanatory, quickly becomes complex when format and storage of digital data are taken into account. In the most generic sense, digital line graphs, triangulated irregular networks (TINs), grids, and Light Detection and Ranging (LiDAR) point clouds are all DEMs. In practical applications, a DEM typically refers to a grid where elevation values are provided at regular intervals in the x and y directions (USGS 1998). We use grid DEM for our discussion, though the issues raised can easily be extended to other types of DEMs.

Similar to "D," the "E" in DEM also has many interpretations. Normally, it refers to the bare-earth elevation void of vegetation and man-made features. However, some applications such as telecommunications and forestry are more concerned with the elevations of the top surfaces. When water bodies are present, elevation could mean underwater depth (i.e. bathymetry) or the elevation of the water surface. Further complicating the issue are the many factors impacting elevation measurement: vertical and horizontal datums, geoid model, coordinate system, units, etc. In this article, we assume these factors have been determined beforehand. Elevation refers to bare-earth elevation and water bodies have the elevation of their surfaces.

In contrast to "D" and "E," the "M" in DEM has received much less attention. Literally, a model is "a schematic description of a system, theory, or phenomenon that accounts for its known or inferred properties and may be used for further study of its characteristics" (*The American Heritage Dictionary of the English Language* 2006). According to this definition, a DEM can be expected to (1) be a schematic description of terrain, (2) be able to account for the known or inferred properties of terrain; and (3) be used to further our understanding of terrain characteristics. Indeed, any DEM is a schematic description of terrain because a DEM is made up by a finite number of points whereas terrain has an infinite number of points. The challenge is how to make a DEM to account for the "known or inferred properties" of terrain. In this article, we outline three terrain properties and discuss their implications for DEM generation. Such a discussion is timely as various DEM generation methods have been reported in the literature (e.g., Hutchinson 1989; Kidner 2003; Barber and Shortridge 2005; Shortridge 2006; Liu et al. 2012). Given that a DEM is

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only a schematic description of terrain, it inevitably contains errors and uncertainties. To help users assess the errors and account for the uncertainties in a DEM, many methods have been developed (e.g., Hodgson and Bresnahan 2004; Hu, Liu, and Hu 2009a; Behrens et al. 2010; Shortridge and Messina 2011; Wilson 2012; O'Neil and Shortridge 2013). However, considering that most DEM users would spend a minimal amount of time assessing DEM uncertainty (Wechsler 2003), a more proactive approach is to control DEM quality during its generation. It is from this perspective that we conducted the research. In "Terrain properties" section, we outline three properties of terrain. The implication of each property for DEM generation is discussed in "Mathematical implications for DEM generation" section. Finally, we make suggestions for generating an optimal DEM in "Implications for LiDAR-derived DEMs" section as well as the priorities for future research in "Conclusion" section.

Terrain properties

We perceive a DEM as a subset of terrain because a DEM consists of only a limited number of points whereas terrain has innumerable points. DEM generation is thus equivalent to building an optimal subset of a terrain. Since an optimal subset is usually obtained by satisfying a set of constraints, a starting point is to identify these constraints and explore the methods to satisfy them. To this end, terrain properties must be understood and articulated.

Terrain may have many known or inferred properties. This article outlines three and discusses their implications for DEM generation. The first property is that each point (x, y) on a topographical surface has a single true elevation denoted by z. The exact value of z may never be known, but this value exists and does not change unless the terrain is modified. Recognition of z as a single, fixed, though possibly unknown, value is important, because it is the basis for assessing DEM vertical accuracy. The second property is that terrain points have an order and sequence determined by their elevations. It is this elevation order and sequence that determines how water flows from one location to another. The third property is that terrain has skeletons or key characteristics; some terrain points form the skeletons, the others do not. In the rest of the article, we explain the implications of each property from a mathematical perspective and explore how to address them during DEM generation.

Mathematical implications for DEM generation

Bijection

The property that each terrain point has a single fixed elevation has two implications for DEM generation; one of them is bijection. As mentioned previously, each point (x, y) has exactly one true elevation z. When a DEM generation function f is applied, it creates an estimated elevation $f(x, y)$. Given that z is a single value, $f(x, y)$ must also be a single value, otherwise it is impossible to assess $z - f(x, y)$, i.e., the vertical error at a point. That z and $f(x, y)$ are both single values suggests there must be a one-to-one relationship between a topographical surface and the surface created by a DEM generation function f. Mathematically, if a function ensures a one-to-one relationship between two sets, it is called a "bijective function" or "bijection." This introduces the first mathematical implication for DEM generation, namely, a DEM generation function f must be bijective.

An examination of DEM generation methods in the literature reveals that not all methods can satisfy this requirement. Some methods can guarantee a one-to-one correspondence at most locations, but they cannot guarantee this correspondence holds *everywhere.* An example is high-order, piecewise, polynomial interpolation that has been used by some studies in the belief that such methods produce more accurate estimates (Kidner 2003; Li, Taylor, and Kidner 2005; Shi and Tian 2006). These methods divide a topographical surface into contiguous and non-overlapping pieces so that interpolation can be conducted piece by piece. Each piece is modeled by a high-order polynomial function. Take the 8-term bicubic interpolation used by Kidner (2003) as an example, this method models each piece of a terrain as a bicubic surface described by Equation (1). The eight coefficients associated with each piece are inferred based on the reference points in that piece alone:

$$f(x, y) = a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{11}xy + a_{02}y^2 + a_{21}x^2y + a_{12}xy^2 \quad (1)$$

Bijection requires that, by using this bicubic interpolation, each point must have exactly one estimated elevation. This requirement is not a problem for most points. However, if a point is on the boundary of two adjacent pieces, its elevation may be estimated by either bicubic piece. Equation (1) cannot guarantee that the two independent estimates are exactly the same, because the coefficients used by each piece are estimated separately. Postprocessing such as averaging the two estimates can force $f(x, y)$ to be a single value, but such processing is ad hoc and lacks theoretical justification. More importantly, as will be shown later in Figure 1, a DEM generation method that forces bijection through postprocessing will not create an isomorphic DEM, which is very important for hydrological and geomorphological applications. Thus, even though high-order polynomial interpolation may result in smaller vertical error, its use in DEM generation warrants caution. In contrast, first-order interpolants such as linear interpolation in 1D, TIN, and bilinear

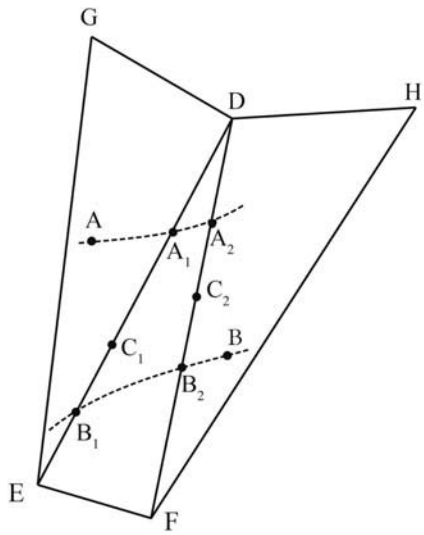

Figure 1. The necessity of bijection in TIN's isomorphism (Hu, Liu, and Hu 2009a). A, A_1, and A_2 are on the same contour line. B, B_1, and B_2 are on another contour line of lower elevation value. Bijection guarantees that a point on a triangle boundary (e.g., A_1, B_1, A_2, B_2, C_1, C_2) has exactly the same estimated elevation based on either triangle. In this way, points on different triangle patches (e.g., A and B) can be compared.

interpolation in a rectangle do not have such a problem. These methods always satisfy the bijection requirement automatically (Hu, Liu, and Hu 2009a).

Vertical accuracy

An essential use of DEMs is to obtain a reliable estimate of elevation values; vertical accuracy is thus an important aspect of DEM generation. The property that each terrain point has a single fixed elevation enables the use of numerical analysis to assess whether a DEM generation method will result in a DEM of desired vertical accuracy. Supposing a terrain surface is $A = \{(x, y, z)\}$, a bijective DEM generation function f creates another surface $A' = \{(x, y, f(x, y))\}$ from which some points $\{(x', y')\}$ are selected to form a DEM$A'' = \{(x', y', f(x', y'))\}$. The selection of $\{(x', y')\}$ is determined by the spatial resolution of the DEM. As DEM spatial resolution varies, every point in A' has a chance to be selected as a grid point. To create a DEM A'' (which is a subset of A') of desired vertical accuracy, the DEM generation function f must ensure that the vertical error at *any* point in A' is within the tolerance threshold.

This introduces the question of how to ensure that the vertical error of A' is acceptable. In the literature, statistical methods are often used. An example is the US National Standard of Spatial Data Accuracy (FGDC 1998), which requires using root mean square error (RMSE) if vertical errors are normally distributed; otherwise, the 95th percentile is used. However, the adequacy of RMSE has been questioned by many studies (e.g., Shortridge 2006; Fisher and Tate 2006; Höhle and Höhle 2009). As pointed out by Liu et al. (2012), RMSE is only effective if vertical errors are random, independent, and identically distributed, which is rarely satisfied in the real world.

A viable alternative is numerical analysis, in particular approximation theory, which is based on calculus instead of statistics. Approximation theory is routinely used in numerical analysis to evaluate the accuracy of approximating a complex function by applying simpler functions and quantitatively characterizing the errors therein (Atkinson and Han 2004). In the context of terrain modeling, a topographical surface can be perceived as a complex mathematical function (A) with an unknown form. A DEM generation function creates another surface (A'), which serves as an approximation of A. According to approximation theory, the accuracy of the approximation is determined by the largest error at *any* point in the entire surface, i.e., $\max\{z - f(x, y)\}$. The rationale behind approximation theory is simple: if the largest error at a point in the *entire* terrain is acceptable, the error at *any* other point must also be acceptable, and A' must thus be an acceptable approximation of A.

To apply approximation theory, each vertical error must be a fixed value. This requirement is satisfied because, as discussed previously, each terrain point has a fixed true elevation. The elevation estimated by a DEM generation function is also fixed. The difference between them, a vertical error, must also be fixed. In general, the vertical error at a point (x, y) can be written as:

$$z - f(x, y) = \delta(x, y) + R(x, y)$$

where $R(x, y)$ is interpolation error and $\delta(x, y)$ is propagation error (Hu, Liu, and Hu 2009b). Interpolation error is due to the imperfectness of a DEM generation function and has nothing to do with the source data quality. Propagation error, on the other hand, is an error in the source data that is propagated to a grid point during DEM generation. Applying approximation theory, the vertical accuracy of a DEM generation function is determined by $\max\{|z - f(x, y)|\}$. $\max\{|z - f(x, y)|\}$ is bounded by $\max|\delta(x, y)| + \max|R(x, y)|$. The values of these terms can be derived if the mathematical form of the DEM generation function is known. Table 1 lists the error bounds of three interpolation methods often used in DEM: linear interpolation in 1D, TIN interpolation, and bilinear interpolation in a rectangle. A case study on how to use the error bounds to assess the vertical accuracy, and thus control the quality, of a DEM created by linear interpolation can be found in Liu et al. (2012). Compared to statistical methods, approximation theory can not only assess the overall accuracy, it can also point out where

Table 1. Vertical error at a terrain point (x, y) (Hu, Liu, and Hu 2009b).

	$\delta(x, y)$: propagation error	$R(x, y)$: interpolation error
Linear interpolation in 1D	$\lvert\delta(x, y)\rvert \le \lvert\delta_{source}\rvert$	$\lvert R(x, y)\rvert \le \frac{1}{8} M_2 h^2$
TIN interpolation	$\lvert\delta(x, y)\rvert \le \lvert\delta_{source}\rvert$	$\lvert R(x, y)\rvert \le \frac{3}{8} M_2 h^2$
Bilinear interpolation ina rectangle	$\lvert\delta(x, y)\rvert \le \lvert\delta_{source}\rvert$	$\lvert R(x, y)\rvert \le \frac{1}{4} M_2 h^2 + \frac{1}{64} M_4 h^4$
Overall vertical accuracy	$\max\lvert\delta(x, y) + R(x, y)\rvert \le \max\lvert\delta(x, y)\rvert + \max\lvert R(x, y)\rvert$	

Note: δ_{source} is the largest error in the source data used to interpolate (x, y); M_2 and M_4 are the maximum norms of second- and fourth-order derivatives, which describe terrain complexity; h is the largest distance between two reference points, which describes sample density.

the user-desired vertical accuracy is not met. Moreover, approximation theory can guide the DEM producer on which areas need additional reference data so as to effectively reduce vertical errors.

Isomorphism

A terrain is made up by a set of points. While the exact elevation at a point is important, the sequence created by ordering or ranking terrain points according to their elevations is equally important. Consider three points a, b, and c, where a is higher than b and b is higher than c. If a drop of water is to pass the three points, it must be from a to b and then to c. When a DEM generation method is applied, each point gets an estimated elevation. It is possible that the vertical errors at all three points are acceptable. However, this does not guarantee that the estimated elevation of a is higher than that of b and c. In other words, the flow direction drawn based on estimated elevations is not always the same as that based on true elevations. This suggests that elevation order and sequence is another important property of terrain. If a DEM is to be mainly used for hydrological and geomorphological applications, its ability to preserve elevation order and sequence is more important than its vertical accuracy.

The ability of a DEM to preserve elevation sequence can be described mathematically as follows: supposing a and b are two points whose true elevations are z_a and z_b, their estimated elevations in a DEM are Z_a and Z_b, respectively. If $z_a < z_b$, does it follow that $Z_a < Z_b$? Similarly, if $Z_a < Z_b$, does it follow that $z_a < z_b$? The answer relates to the concept of *isomorphism* in set theory. The word *isomorphism* is from the Greek language and literally means "equal shape." It is studied in mathematics in order to extend insights from one set to another. If two sets are isomorphic, then any property true to one set is also true to the other. In the context of terrain modeling, let a and b be two grid points in a DEM A'', i.e., $a \in A''$ and $b \in A''$. Since a DEM A'' is a subset of A', which is the surface created by a DEM generation function f, i.e., $A'' \subset A'$, there are $a \in A'$ and $b \in A'$. Supposing a is lower than b in A', i.e., $Z_a < Z_b$, an isomorphic DEM generation function f guarantees that $z_a < z_b$. In other words, if the DEM suggests that the flow direction is from a to b, then, in theory, this must be true in the field. In reality, the accuracy of a flow direction derived from an isomorphic DEM may be compromised by the limitations in the existing flow-direction algorithms and the discretization of the landscape (O'Neil and Shortridge 2013). Nevertheless, an isomorphic DEM generation function provides the theoretical basis for generating accurate drainage networks and geomorphology features.

In order for a DEM generation method to be an isomorphism, it must meet two requirements (Hu, Liu, and Hu 2009a). The first is the ability to divide a terrain into a set of contiguous monotonic patches with no "bumps" or "dips," so that each patch can be reasonably modeled as a smooth facet. TIN and linear interpolation in 1D can both meet this requirement. However, high-order polynomial interpolation such as the bicubic interpolation discussed previously does not, because features such as ridges and channels do not always fall on an edge of a bicubic patch. Increasing the spatial resolution in these methods will increase the likelihood for a patch to be monotonic, but the problem cannot be eliminated. This limitation suggests that high-order interpolations are not suitable for creating DEMs for hydrological or geomorphologic applications.

In addition to monotonic patching, another requirement for creating an isomorphic DEM is that the DEM generation function is a bijection (see earlier). This is illustrated in Figure 1 using TIN as an example. Supposing point A on the terrain is higher than point B, i.e., $z_A > z_B$. If A and B are both on the same patch, and the patch is monotonic, as in the case of TIN, $z_A > z_B$ entails $Z_A > Z_B$ (Hu, Liu, and Hu 2009a). However, what if A and B are located on nonadjacent patches? As illustrated in Figure 1, Z_A is estimated based on patch GDE while Z_B is estimated based on patch DFH. To enable a comparison of the two points, bijection is necessary. Because in Figure 1 GDE, DEF, and DFH are monotonic triangle patches, there must be a point A_1 on the edge of patch GDE so that $z_A = z_{A_1}$. Similarly, there must be A_2 for $z_{A_1} = z_{A_2}$. A, A_1, and A_2 could be three points from the same contour line that crosses the three triangular patches. By the same token, B_1 and B_2 must exist to get $z_{B_1} = z_{B_2} = z_B$. If $z_A > z_B$, it is easy to find two points C_1 and C_2 such that

$z_{A_1} > z_{C_1} > z_{C_2} > z_B$. Given that A and C_1 are both on patch EDG, $z_A > z_{C_1}$ entails $Z_A > Z_{C_1}^{EDG}$ where $Z_{C_1}^{EDG}$ is the estimated elevation of C_1 based on patch EDG. $Z_{C_1}^{DEF} > Z_{C_2}^{DEF}$ and $Z_{C_2}^{DFH} > Z_B$ can be derived similarly. Bijection becomes critical hereupon. If a DEM generation function is a bijection (as in the case of TIN), then $Z_{C_1}^{EDG} = Z_{C_1}^{DEF}$ and $Z_{C_2}^{DEF} = Z_{C_2}^{DFH}$. Previous comparisons based on individual patches can then be combined, i.e.,

$$Z_A > \left(Z_{C_1}^{EDG} = Z_{C_1}^{DEF}\right) > \left(Z_{C_2}^{DEF} = Z_{C_2}^{DFH}\right) > Z_B \ \Rightarrow \ Z_A > Z_B$$

If, however, a DEM generation function is not a bijection, the estimate of C_1 based on patch EDG will not be exactly the same as the estimate of C_1 based on patch DFH, i.e., $Z_{C_1}^{EDG} \neq Z_{C_1}^{DEF}$. Consequently, $Z_A > Z_{C_1}^{EDG}$ and $Z_{C_1}^{DEF} > Z_{C_2}^{DEF}$ cannot be combined and the comparison in the intermediate patches cannot proceed. Hu, Liu, and Hu (2009a) examined the ability of selected interpolation methods to meet the isomorphism requirement (Table 2). Other DEM generation methods may be studied in a similar manner.

Generalization

A terrain has an infinite number of points, but they do not play the same roles in delineating the key characteristics of the terrain. Those that are local and global extrema usually define the basic structure of a terrain. For example, peaks are local maxima that are higher than all neighbors. Pits, on the other hand, are local minima. Saddles are not extrema but are important because they denote where a terrain surface curves up in one direction while curving down in another direction. Ridges and channels are lines of divergent and convergent slopes, respectively. Peaks, pits, passes, ridges, and channels form the basic structure of a terrain. They must be retained with high fidelity during DEM generation if the resultant DEM is to be used to study terrain structure and landforms in geomorphology, for instance.

Table 2. The ability of some interpolation methods to ensure isomorphism.

	Isomorphism assured?
Linear interpolation in 1D	Yes, proactively
TIN interpolation	Yes, but only if all break points and break lines are incorporated during TIN construction
Bilinear interpolation in a rectangle	No, because rectangular pieces are not always monotonic
High-order polynomial interpolation functions	No, because they are not bijective

Theoretically, if a DEM generation function f is an isomorphism, each point in the terrain corresponds to exactly one point in surface $A' = \{(x, y, f(x, y)\}$. Furthermore, because isomorphism guarantees that higher-lying points in terrain remain higher in A', A' must preserve all terrain features. The challenge is that a DEM A'' is only a subset of A', and DEM points $\{(x', y')\}$ are placed only at designated locations. Critical features such as peaks and pits rarely fall exactly at designated locations and can thus be easily lost unless there is displacement or shifting. On the other hand, once the spatial resolution is determined, the number of points in a grid DEM is fixed, meaning that a DEM may not be able to include *all* structure-defining points. Selection is thus necessary. This presents an even bigger challenge because selection is scale-dependent. A DEM contains fewer and fewer grid points as its cell size becomes bigger, meaning that more terrain points have to be represented by a single grid point in a DEM. These two aspects lead to the question: which single point should be selected and where should this selected point be placed in a DEM?

The question leads to the concept of generalization. In cartography, generalization refers to “the selection and simplified representation of detail appropriate to the scale and/or purpose of the map” (ICA 1973). However, the concept of generalization is not limited to cartography or DEMs. As pointed out by Brassel and Weibel (1988, 230), “generalization as an abstraction process is central to inductive scientific reasoning and underlies all modelling in science, art, and technology.” Since a DEM is a model, it must involve removing the “unimportant” and focusing on the crucial elements of terrain. From this perspective, generalization is mandatory in any DEM generation.

Regrettably, the necessity of generalization has not been fully recognized in the DEM literature. This is reflected by how DEM values are interpreted. As summarized by Longley et al. (2005, 327), “The elevation of a grid cell is often the elevation of the cell’s central point”, though “sometimes it is the mean elevation of the cell, and other rules have been used to define the cell’s elevation.” A grid cell contains innumerable terrain points. If it is always the central point that is selected to represent a grid cell, important terrain features that are not at cell centroids will inevitably be lost. Similarly, if the elevation of a grid cell is always the average elevation of the cell, terrain will be increasingly smoothed out as cell size increases. This is illustrated in Figure 2 where several DEMs are created by using the average elevation of a cell. It can be seen that, as cell size increases from 3 to 100 m, the corresponding DEM becomes flatter. Clearly, for the study area in Figure 2 which has significant elevation variation, it is not appropriate to create a DEM using the average-elevation method.

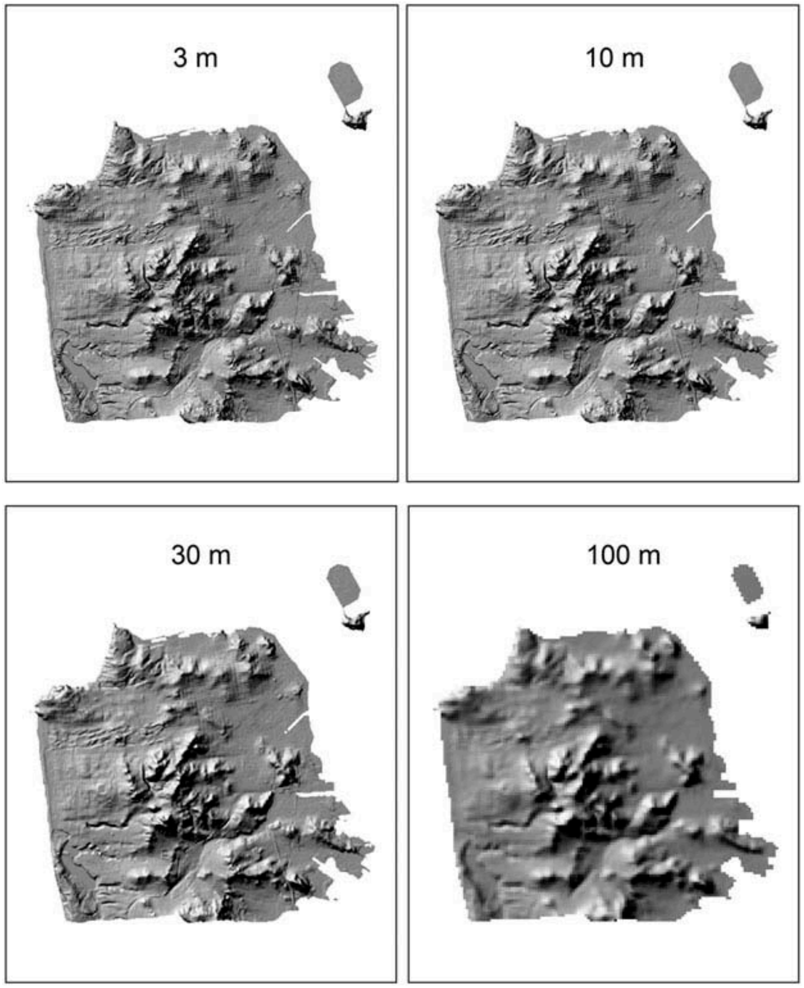

Figure 2. Mechanistic generalization by using statistical mean as the value of a cell. Terrain becomes increasingly smoother out as cell size increases.

If neither the statistical mean nor the centroid's elevation is appropriate to be the value of a grid cell, then what should be used? A cell contains innumerable points, generalization simplifies them into a single point. From this perspective, a cell can be perceived as this point's region of influence. In Figure 3, *a* to *i* are nine grid points in a DEM and each influences or controls its corresponding cell. If a new DEM of three times the cell size is to be generated, these nine points must be simplified to point *o*. Point *o* coincides with point *e* but its region of influence is the combination of all nine cells. Since *o* is the only point in the region to be included in a DEM, and it is allowed to have only one elevation value, *o* should carry the most significant information about the terrain in its region of influence. To this end, the most significant terrain point in the region must be identified and shifted to point *o*.

In general, terrain points can be classified into three groups according to their information richness: critical points such as peaks, pits, and saddles; special points located on a ridge or a valley; and ordinary points. If no critical or special points exist in a cell, i.e., if the corresponding terrain is flat with no "bumps" or "dips," the elevation of *o* can simply be the average elevation of the cell. However, if a ridge or channel crosses the cell, special points exist. Because special points reveal more information about terrain than ordinary points, they should be selected and shifted to a grid point. When there are multiple special and critical points, they must be ranked according to their significance in the overall terrain. The word "overall" is necessary as the significance of a terrain feature is scale-dependent. For example, a peak is usually considered more significant than an ordinary point. However, if the peak is located on a large syncline, its significance weakens as DEM cell size increases. Many algorithms are available to extract terrain features, e.g., Peucker and Douglas (1975) and Tarboton, Bras, and Rodriguez-Iturbe (1991). The challenge is how to rank them and displace them in a way that preserves their elevation sequence and order. By now,

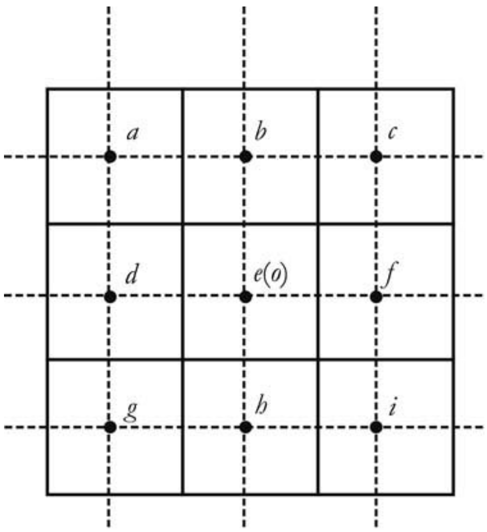

Figure 3. A DEM point's region of influence. Grid points *a–i* have the grid cell centered on them as their region of influence. After generalization, the nine cells are aggregated to a single cell. This single cell is the region of influence of point *o*.

several efforts on DEM generalization have been reported. The majority of them are based on the 3-D Douglas–Peucker algorithm or its equivalent (Fei and He 2009; Ai and Li 2010). Since none of these methods has been proved isomorphic, i.e., be able to preserve the original elevation sequence and order in the resultant DEM, generalization remains challenging.

Implications for LiDAR-derived DEMs

The previous section explained the implications of terrain properties for DEM generation. This section discusses them in the context of LiDAR-derived DEMs. Advancement in airborne laser technology has increased the availability of LiDAR-acquired elevation data (Yan et al. 2012). LiDAR collects data in the form of high-density mass points. If bare-earth elevation is extracted accurately, mass points can provide a terrain model that has very high vertical accuracy and preserves terrain structures well. However, many end users of LiDAR do not need points of very high density; they have concerns about redundancy, cost of storage, and processing time. Furthermore, end users usually want a grid instead of mass points so that topography can be generated in existing Computer Aided Design and/or Geographical Information Systems environments (Maune et al. 2007). A typical approach to creating a grid is to reduce the number of mass points, build a TIN from the remaining points, and then interpolate a grid from the TIN. Given the implications discussed earlier, a couple of aspects need to be considered.

The first is how to select which mass points to retain, a decision closely related to the task of generalization. The other is the TIN-to-grid interpolation. A main driver behind the adoption of LiDAR technology is hydrological analysis (e.g., flood mitigation) in relatively flat areas. As discussed earlier, isomorphism is critical to ensure a DEM's hydrological fidelity. A TIN is isomorphic mathematically, but only if the TIN is constructed by incorporating *all* break lines and break points (Hu, Liu, and Hu 2009a). Break lines and break points are scale-dependent. It is very difficult, if not impossible, to identify all break lines and break points at a specified scale. Without an isomorphic TIN, the grid interpolated from the TIN will not be isomorphic either. Thus, although LiDAR offers an engineering solution to many challenges in DEM generation, the challenges of isomorphism and generalization remain. Solutions to these challenges are crucial to fulfill LiDAR's potential for creating DEMs with contour accuracy to 2 feet in most of the terrain and land cover types (FEMA 2003).

Generalization also offers a mathematical solution to creating multi-resolution DEMs from a LiDAR point cloud. Geospatial research usually involves data collected at varying scales. These data have to be converted to the same scale before they can be integrated. Currently, the US National Elevation Dataset only provides nationwide DEMs at 1/9-, 1/3-, and 1-arc second. If DEMs of other resolutions are desired, they can be produced using photogrammetry or remote sensing technology; however, the cost can be prohibitive. Generalization provides a mathematical alternative. In fact, the very high density and the very high vertical accuracy of LiDAR mass points provide a nearly perfect basis for generalization, as one can assume that nearly all terrain features have been well captured by the mass points. Generalization would enable the generation of a DEM at any spatial resolution. If generalization can be completed on the fly, DEMs of varying resolutions will not need to be stored separately, as they now do. Only bare-earth LiDAR points will need to be stored, updated, and maintained. From this perspective, isomorphism and generalization should become the highest priority in LiDAR-DEM research.

Conclusion

DEMs were developed to enable automatic analysis of the earth's topography and reduce the need for labor-intensive interpretation (Maune et al. 2007). To fulfill this expectation, a DEM must be able to account for the properties of terrain. Existing digital terrain research has mainly focused on the DEMs themselves. A close examination of terrain properties – the starting point for building a model – is still lacking. It is from this perspective that we examined three properties of terrain and discussed their four implications for DEM generation. The first property is that each terrain point has a single, fixed, though possibly unknown, elevation. This property determines that a DEM generation method must be a bijection in order to guarantee a one-to-one relationship between a topographical surface and the surface corresponding to the DEM generation function. This property also enables the application of approximation theory from numerical analysis to assessing whether a DEM generation method is able to create a DEM of desired vertical accuracy. The second property of terrain is that there is an order and sequence among terrain points. This order and sequence is created by ranking terrain points according to their elevations. If a DEM will be mainly used for hydrological or geomorphological purposes, the emphasis should be on the preservation of the elevation sequence, not vertical accuracy. To preserve the elevation sequence, a DEM generation function must be an isomorphism, which would guarantee (at least in theory) that higher points in the terrain remain higher in the DEM and higher points in the DEM are indeed higher in the terrain. Finally, terrain points play different roles in delineating the structures of a terrain: some form terrain skeletons and others do not. If a DEM is to preserve the terrain skeletons, the DEM generation method must involve generalization by identifying and shifting information-rich points. An ideal DEM generation method would simultaneously satisfy the requirement of bijection, minimal vertical error, isomorphism, and generalization. However, existing research evaluates DEM generation methods mainly from the vertical accuracy perspective. As LiDAR-acquired elevation data become increasingly available, vertical error should no longer be the dominant focus of research. Instead, isomorphism and generalization should receive attention in future research.

References

Ai, T., and J. Li. 2010. "A DEM Generalization by Minor Valley Branch Detection and Grid Filling." *ISPRS Journal of Photogrammetry and Remote Sensing* 65: 198–207. doi:10.1016/j.isprsjprs.2009.11.001.

The American Heritage Dictionary of the English Language. 4th ed. 2006. Boston, MA: Houghton Mifflin.

Atkinson, K., and W. Han. 2004. *Elementary Numerical Analysis*. 3rd ed. Hoboken, NJ: John Wiley & Sons.

Barber, P., and A. Shortridge. 2005. "Lidar Elevation Data for Surface Hydrologic Modeling: Resolution and Representation Issues." *Cartography and Geographic Information Science* 32: 401–410. doi:10.1559/152304005775194692.

Behrens, T., A. Zhu, K. Schmidt, and T. Scholten. 2010. "Multi-Scale Digital Terrain Analysis and Feature Selection for Digital Soil Mapping." *Geoderma* 155 (3–4): 175–185. doi:10.1016/j.geoderma.2009.07.010.

Brassel, K., and R. Weibel. 1988. "A Review and Conceptual Framework of Automated Map Generalization." *International Journal of Geographical Information Systems* 2 (3): 229–244. doi:10.1080/02693798808927898.

FEMA (Federal Emergency Management Agency). 2003. "Guidelines and Specifications for Flood Hazard Mapping Partners, Appendix A: Guidance for Aerial Mapping and Surveying." Accessed February 11, 2014. http://www.fema.gov/media-library-data/20130726-1543-20490-2653/frm__gsaa.pdf

FGDC (Federal Geographic Data Committee). 1998. "FGDC-STD-007.3-1998, Geospatial Positioning Accuracy Standards, Part 3: National Standard for Spatial Data Accuracy." Accessed February 11, 2014. https://www.fgdc.gov/standards/projects/FGDC-standards-projects/accuracy/part3/chapter3

Fei, L., and J. He. 2009. "A Three-Dimensional Douglas-Peucker Algorithm and Its Application to Automated Generalization of Dems." *International Journal of Geographical Information Science* 23: 703–718. doi:10.1080/13658810701703001.

Fisher, P. F., and N. J. Tate. 2006. "Causes and Consequences of Error in Digital Elevation Models." *Progress in Physical Geography* 30 (4): 467–489. doi:10.1191/0309133306pp492ra.

Hodgson, M., and P. Bresnahan. 2004. "Accuracy of Airborne-Derived Elevation: Empirical Assessment and Error Budget." *Photogrammetric Engineering and Remote Sensing* 70 (3): 331–340. doi:10.14358/PERS.70.3.331.

Höhle, J., and M. Höhle. 2009. "Accuracy Assessment of Digital Elevation Models by Means of Robust Statistical Methods." *ISPRS Journal of Photogrammetry and Remote Sensing* 64: 398–406. doi:10.1016/j.isprsjprs.2009.02.003.

Hu, P., X. Liu, and H. Hu. 2009a. "Isomorphism in Digital Elevation Models and Its Implication to Interpolation Functions." *Photogrammetric Engineering and Remote Sensing* 75: 713–721. doi:10.14358/PERS.75.6.713.

Hu, P., X. Liu, and H. Hu. 2009b. "Accuracy Assessment of Digital Elevation Models Based on Approximation Theory." *Photogrammetric Engineering and Remote Sensing* 75: 49–56. doi:10.14358/PERS.75.1.49.

Hutchinson, M. F. 1989. "A New Procedure for Gridding Elevation and Stream Line Data with Automatic Removal of Spurious Pits." *Journal of Hydrology* 106 (3–4): 211–232. doi:10.1016/0022-1694(89)90073-5.

ICA (International Cartographic Association). 1973. *Multilingual Dictionary of Technical Terms in Cartography*. Wiesbaden: Franz Steiner (Verlag GmbH).

Kidner, D. B. 2003. "Higher-Order Interpolation of Regular Grid Digital Elevation Models." *International Journal of Remote Sensing* 24: 2981–2987. doi:10.1080/0143116031000086835.

Li, J., G. Taylor, and D. Kidner. 2005. "Accuracy and Reliability of Map-Matched GPS Coordinates: The Dependence on Terrain Model Resolution and Interpolation Algorithm." *Computers & Geosciences* 31: 241–251. doi:10.1016/j.cageo.2004.06.011.

Liu, X., P. Hu, H. Hu, and J. Sherba. 2012. "Approximation Theory Applied to DEM Vertical Accuracy Assessment." *Transactions in GIS* 16: 393–410. doi:10.1111/j.1467-9671.2012.01343.x.

Longley, P., M. Goodchild, D. Maguire, and D. Rhind. 2005. *Geographical Information Systems and Science*. 2nd ed. Chichester: John Wiley & Sons.

Maune, D., S. Kopp, C. Crawford, and C. Zervas. 2007. "Introduction." In *Digital Elevation Model Technologies and Applications: The DEM Users Manual*, 2nd ed., edited by D. Maune, 1–35. Bethesda, MD: American Society of Photogrammetry and Remote Sensing.

O'Neil, G., and A. Shortridge. 2013. "Quantifying Local Flow Direction Uncertainty." *International Journal of Geographical Information Science* 27: 1292–1311. doi:10.1080/13658816.2012.719627.

Peucker, T., and D. Douglas. 1975. "Detection of Surface-Specific Points by Local Parallel Processing of Discrete Terrain Elevation Data." *Computer Graphics and Image Processing* 4 (4): 375–387. doi:10.1016/0146-664X(75)90005-2.

Shi, W., and Y. Tian. 2006. "A Hybrid Interpolation Method for the Refinement of A Regular Grid Digital Elevation Model." *International Journal of Geographical Information Science* 20 (1): 53–67. doi:10.1080/13658810500286943.

Shortridge, A. 2006. "Shuttle Radar Topography Mission Elevation Data Error and Its Relationship to Land Cover." *Cartography and Geographic Information Science* 33: 65–75. doi:10.1559/152304006777323172.

Shortridge, A., and J. Messina. 2011. "Spatial Structure and Landscape Associations of SRTM Error." *Remote Sensing of Environment* 115 (6): 1576–1587. doi:10.1016/j.rse.2011.02.017.

Tarboton, D., R. Bras, and I. Rodriguez-Iturbe. 1991. "On the Extraction of Channel Networks from Digital Elevation Data." *Hydrological Processes* 5: 81–100. doi:10.1002/hyp.3360050107.

USGS. 1998. "Standards for Digital Elevation Models." Accessed September 2, 2014. http://nationalmap.gov/standards/pdf/PDEM0198.PDF

Wechsler, S. 2003. "Perceptions of Digital Elevation Model Uncertainty by DEM Users." *URISA Journal* 2: 57–64.

Wilson, J. P. 2012. "Digital Terrain Modeling." *Geomorphology* 137 (1): 107–121. doi:10.1016/j.geomorph.2011.03.012.

Yan, M., T. Blaschke, Y. Liu, and L. Wu. 2012. "An Object Based Analysis Filtering Algorithm for Airborne Laser Scanning." *International Journal of Remote Sensing* 33 (22): 7099–7116. doi:10.1080/01431161.2012.699694.