SFSU – ENGR 445 – ANALOG IC DESIGN LAB

LAB #5: FREQUENCY RESPONSE OF BJT AMPLIFIERS
(Updated Dec. 23, 2002)

Objective:
To investigate the frequency response of basic BJT amplifier configurations. To characterize a BJT dynamically. To compare measurements with computer simulations.

Components:
2 × 2N2222 npn BJTs, 1 × 1N4148 diode, 3 × 0.1µF capacitors, 1 × 10µF capacitor, 1 × 10 kΩ pot, and resistors: 1 × 10 Ω, 3 × 1.0 kΩ, 4 × 10 kΩ, and 1 × 1.0 MΩ (all 5%, ¼ W).

Instrumentation:
A dual adjustable regulated power supply, a digital multi-meter (DMM), a signal generator (sine wave, square wave), and a dual-trace oscilloscope.

PART I – THEORETICAL BACKGROUND

The dynamic characteristics of a BJT are controlled by three capacitive components:

- $C_{je}$, the base-emitter (B-E) junction capacitance
- $C_{jc}$, the base-collector (B-C) junction capacitance, usually denoted as $C_\mu$ in the analog literature
- $C_b$, the base-charging capacitance

For junction-isolated monolithic BJTs there is also a fourth capacitive component, namely, the collector-to-substrate capacitance $C_{js}$. Figure 1 shows the location of $C_{je}$, $C_b$, and $C_\mu$ in the BJT’s small-signal model. For a BJT of the npn type, the expressions for the B-E and B-C junction capacitances are

$$C_\mu = \frac{C_{\mu 0}}{(1-v_{BE}/\phi_e)^{m_e}}$$
$$C_{je} = \frac{C_{je 0}}{(1-v_{BC}/\phi_c)^{m_c}}$$

where:
- $C_{je 0}$ and $C_{\mu 0}$ are the zero-bias values of $C_{je}$ and $C_\mu$
- $\phi_e$ and $\phi_c$ are the built-in potentials of the B-E and the B-C junctions
- $m_e$ and $m_c$ are the grading coefficients of the B-E and B-C junctions
- $v_{BE}$ and $v_{BC}$ are the B-E and B-C voltage drops

Fig. 1 – Small signal BJT model.
The base-charging capacitance $C_b$ depends on the dc bias of the BJT as

$$C_b = \tau_f g_m$$

where:
- $\tau_f$ is the forward base transit time
- $g_m$ is the transconductance
- $W_B$ is the effective base-width
- $D_b$ is the diffusion constant of minority carriers in the base
- $I_C$ is the collector dc bias current
- $V_T$ is the thermal voltage ($V_T = 26$ mV at room temperature)

Since $C_{je}$ and $C_b$ are in parallel with each other, they are conveniently lumped together as a single equivalent capacitance $C_\pi$,

$$C_\pi = C_{je} + C_b$$

The maximum useful frequency of operation of a BJT is the frequency at which the small-signal current gain $\beta(jf)$ drops to unity. Aptly called the transition frequency $f_T$, it is expressed as

$$f_T = \frac{1}{2\pi} \frac{g_m}{C_\pi + C_\mu}$$

The above capacitances affect the dynamics of a BJT amplifier in different ways, depending on the particular configuration. As a rule, the number of reactive elements in an amplifier (capacitors in the present case) determines the number of poles in its gain $a(s)$, and thus the order of the system. Moreover, depending on circuit topology, gain may admit also zeros. For a physical system, the number $n_z$ of zeros and the number $n_p$ of poles are such that $n_z \leq n_p$. Thus, with two net capacitances, namely, $C_\pi$ and $C_\mu$, a BJT is inherently a 2nd-order system. In transistor circuits of practical interest the response is often dominated by just one pole at $s = -\omega_p$, indicating that gain can be approximated as

$$a(s) \approx \frac{a_0}{1 + s/\omega_p}$$

where $a_0$ is the low-frequency gain. The significance of the pole is two fold, depending on whether we are investigating the circuit’s frequency response or transient response.

- In the frequency domain, depicted in Fig. 2a, $\omega_p$ represents the $-3$-dB frequency of the ac gain. This frequency is found experimentally by driving the circuit with a sine wave of constant-amplitude and variable-frequency, and then finding the frequency $\omega_{-3dB}$ at which the output amplitude drops to $1/\sqrt{2}$, or 70.7% of its low-frequency value. Alternatively, it can be found as the frequency $\omega_{45^\circ}$ at which the output reaches a delay of $-45^\circ$ with respect to the input.

- In the time domain, depicted in Fig. 2b, $1/\omega_p$ represents the time-constant governing the exponential transient at the output. This constant is found experimentally by driving the circuit with a square wave, and then finding the time $t_{63\%}$ at which the output has completed $(1 - 1/e) \times (V_\infty - V_0)$, or 63% of the entire transition. Alternatively, it can be found as the time $t_{\text{intercept}}$ at which the tangent to the transient at $V_0$ (the initial value) intercepts $V_\infty$ (the steady-state value).
The Common-Emitter Amplifier

Of particular significance is the common-emitter (CE) configuration of Fig. 3a, which is conveniently analyzed using the Miller approximation. Such an approximation results in the simplified ac equivalent of Fig. 3b, where

$$ C_t = C_\pi + C_M $$

is called the total equivalent capacitance between base and emitter, and

$$ C_M = (1 + g_mR_o)C_\mu $$

is the Miller capacitance. In the above expression,

$$ R_o = R_C/r_o $$

is the amplifier’s output resistance. Here, $r_o = V_A/I_C$, where $V_A$ is the Early voltage of the BJT. The small-signal voltage gain of the CE configuration is readily found to be

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![Fig. 3](image)

**Fig. 3**– (a) Common-emitter (CE) ac equivalent, and (b) its small-signal model using the Miller approximation.
\[ a(jf) \cong \frac{a_0}{1 + jf / f_b} \]  

(9)

where \( a_0 \) is the low-frequency gain and \( f_b \) is the –3-dB frequency,

\[ a_0 = \frac{r_\pi}{R_s + r_b + r_\pi} \left( -g_m R_o \right) \]  

(10a)

\[ f_b = \frac{1}{2\pi R_i C_i} \]  

(10b)

In the above expressions, \( r_\pi = \beta_0 g_m = \beta_0 V_T / I_C \), where \( \beta_0 \) is the low-frequency small-signal current gain; \( r_b \) is the bulk resistance of the base region; \( R_i \) is the equivalent resistance seen by \( C_i \), or

\[ R_i = (R_s + r_b) / r_\pi \]  

(11)

The gain-bandwidth product is defined as \( GBP = |a_0 \times f_b| \). For the CE amplifier, its expression is

\[ GBP = |a_0 \times f_b| = \frac{g_m R_o}{2\pi(R_s + r_b)C_i} \]  

(12)

**PART II – EXPERIMENTAL PART**

In this lab we are going to investigate the frequency responses of the basic BJT configurations using the 2N2222 discrete npn BJT. The advantage of working with this particular BJT is that its model is already available in PSpice’s Library, so you can always run PSpice simulations to anticipate what to expect in the lab. Also, you are encouraged to compare the findings for your particular 2N2222 sample with the typical data tabulated in the data sheets. The latter can be downloaded from the Web (for instance, by visiting [http://www.google.com](http://www.google.com) and searching for “2N2222” or variants thereof.)

As you assemble your circuits, keep leads short, bypass all power supplies with 0.1-\( \mu \)F capacitors mounted closely to the circuit under investigation, and don’t forget to turn power off whenever you make any circuit changes. Also, to reduce dynamic loading of your circuit by the oscilloscope, use low input-capacitance probes, such as X10-probes.

Henceforth, steps shall be identified by letters as follows: C for calculations, M for measurements, and S for SPICE simulation.

**Preliminary BJT Characterization**

We begin with some preliminary BJT characterizations, whose results will be needed subsequently. Thus, mark one of the 2N2222 BJTs in your kit, and proceed as follows.

**MCI:** With power off, assemble the circuit of Fig. 4 (implement the 5-kΩ resistor using \( 2 \times 10 \) kΩ resistors in parallel). Keep leads short, and bypass the power-supply to ground via a 0.1-\( \mu \)F capacitor, as recommended in the Appendix. Then, apply power, and while monitoring \( I_C \) with the digital current meter (DCM), adjust the potentiometer until \( I_C = 1.0 \) mA. Next, short out \( R_C \) with a wire (that is, close SW) so as to effect the change \( \Delta V_{CE} = 5 \) V and record the corresponding change \( \Delta I_C \) (this change will be small, so use as many digits as your instrument will allow). Finally, compute \( r_o = \Delta V_{CE} / \Delta I_C \).
MC2: With power off, assemble the circuit of Fig. 5, keeping leads short and bypassing both supplies to ground via two 0.1-\(\mu\)F capacitors. Apply power, and while monitoring \(I_C\) with the DCM inserted in series at node \(C\), adjust the dual tracking supplies until \(I_C = 1.0\) mA. Next, shut off power, insert the DCM in series at node \(B\), and measure \(I_B\). Finally, calculate \(\beta = I_C/I_B\), \(g_m = I_C/V_T\), and \(r_x = \beta g_m\).

The Common Emitter (CE) Configuration

Next, we turn to CE amplifier of Fig. 6. Here, we use the voltage divider made up of \(R_1\) and \(R_2\) for the dual purpose of scaling down the signal generator \(v_s\) to a signal \(v_i = v_s R_2/(R_1 + R_2) = v_s/99\) that meets the small-signal requirements of the CE amplifier, and to ensure a low and predictable source resistance.

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RS = R1//R2 ≅ 10 Ω. Moreover, to ensure a good-quality ac ground at the emitter over a broad range of frequencies, we use the parallel combination of a 0.1-μF disc capacitor and a 10-μF polarized capacitor. Polarized capacitors tend to exhibit significant inductive behavior at high frequencies, so it is always good practice to parallel a polarized type with a disc type to achieve a composite device capable of ensuring a good ac ground over a wide range of frequencies.

We observe that our amplifier will exhibit a high-frequency pole at $f_b$, as predicted by Eq. (10b), but also a low-frequency pole at $f_a = 1/(2\pi R_e C_e)$, where $R_e$ is the equivalent resistance seen looking into the emitter, and $C_e$ is the net emitter capacitance ($C_e = 10 + 0.1 ≅ 10$ μF). It is thus understood that in the present context what is referred to as the low-frequency gain $a_0$ of Eq. (10a) denotes the gain over the frequency range $f_a < f < f_b$. For obvious reasons, this gain is also called the mid-band gain, and $f_a$ and $f_b$ the lower and upper pole-frequencies.

MC3: With power off, assemble the circuit of Fig. 6, keeping leads short and with the ground side of the capacitors connected directly to the ground side of $R_2$, as shown. Also, be sure that the “+” plate of the polarized capacitor goes to ground, and the “−” plate to the emitter, which is at a negative voltage. Next, adjust the waveform generator so that $v_s$ is a 100-kHz sine wave with 0-V DC offset and 1-V peak-to-peak amplitude (this makes the amplitude of $v_i$ about 10-mV peak-to-peak, small enough to guarantee the validity of the small-signal model). To reduce dynamic loading of your circuit by the oscilloscope, use low input-capacitance probes, such as X10-probes. Then proceed as follows:

- Apply power, and while monitoring $v_s$ and $v_o$ with Ch. 1 and Ch. 2 of the oscilloscope, both channels set on AC, vary the generator’s frequency up and down the frequency spectrum to observe that gain is fairly constant at mid frequencies, but rolls off both at the low and high ends of the spectrum.
- Measure the mid-band gain from $v_i$ to $v_o$, and multiply it by 99 to obtain the amplifier’s mid-band gain $a_0$ from $v_i$ to $v_o$.
- Lower the frequency until $v_o$ drops to 70.7% of its mid-band value. This is the aforementioned
lower pole frequency $f_a$. Record its experimental value, and compare with its theoretical value $f_a = 1/(2\pi R_C \mu)$. Are they close enough? Account for any possible differences.

- Raise the frequency until $v_o$ again drops to 70.7\% of its mid-band value. This is the upper pole frequency $f_b$. Record its experimental value, and compare with its theoretical value predicted by Eq. (10b). Are they close enough? Account for any possible differences.
- Using the experimental values of $a_0$ and $f_b$, calculate $GBP = |a_0 \times f_b|$.

S4: Simulate the circuit of Fig. 6 via PSpice using the 2N2222 model available in PSpice’s Library. Hence, use the cursor facility of PSpice to determine $a_0$, $f_a$, and $f_b$. Considering that your measurements are based on a particular 2N2222 sample, while the PSpice model is based on typical 2N2222 data, comment on the degree of agreement between measurements and simulation.

MC5: With power off, set $R_B = 0$ in the circuit of Fig. 6 by replacing $R_B$ with a plain wire. Reapply power, and retrace the procedure of Step MC3 to find the new values of $a_0$, $f_b$, and $GBP$. Compare with those of Step MC3, and comment on your findings.

**Note:** With $R_B = 0$, $a_0$ and $f_b$ will increase. Should $f_b$ exceed the upper frequency limit of your waveform generator, you can estimate it via time-domain techniques, as illustrated in Fig. 2b. That is, you first measure the time constant $\tau$ governing the transient response, and then calculate $f_b = 1/(2\pi \tau)$.

S6: Simulate the circuit of Step MC5 via PSpice, and comment on the degree of agreement between measurements and simulation.

C7: Steps MC3 and MC5 are designed to exploit Eq. (12) to establish two equations in the two unknowns $r_b$ and $C_\pi$ (the remaining parameters can be calculated directly), namely,

$$GBP_1 = \frac{g_m R_o}{2\pi(R_s + R_B + r_b)C_\pi}$$  \hspace{1cm} (13a)

$$GBP_2 = \frac{g_m R_o}{2\pi(R_s + r_b)C_\pi}$$  \hspace{1cm} (13b)

where $GBP_1$ is the gain-bandwidth product obtained in Step MC3 with $R_B = 1 \text{k}\Omega$, and $GBP_2$ is that obtained in Step MC5 with $R_B = 0$. Moreover, $R_B \approx 10 \text{\Omega}$, and the product $g_m R_o$ is readily calculated. Thus, solve Eqs.(13) to estimate $r_b$ and $C_\pi$. Are their values typical?

**Hint:** First, take the ratio $GBP_2/GBP_1$, and solve for $r_b$. Then, back substitute into Eq. (13b) and solve for $C_\pi$.

MC8: With power off, assemble the circuit of Fig. 7 (implement the 11-k\Omega resistor as 10 k\Omega in series with 1.0 k\Omega.) Then, turn power on, and measure the new value of $f_b$.

Note that this circuit is the same as that of Fig. 6, except for the modified resistive network seen by the collector, which is designed to change the value of $R_o$ while leaving everything else the same, including the bias conditions of the collector and thus the value of $C_\mu$. Consequently, we now have two equations in the two unknowns $C_\pi$ and $C_\mu$ (the remaining parameters can be calculated directly), namely,

$$f_{b1} = \frac{1}{2\pi R_s[C_\pi + (1 + g_m R_o)]C_\mu}$$  \hspace{1cm} (14a)

$$f_{b2} = \frac{1}{2\pi R_s[C_\pi + (1 + g_m R_o)C_\mu]}$$  \hspace{1cm} (14b)
Here, \( f_{b1} \) is the experimental –3-dB frequency of the circuit of Fig. 6, whose output resistance is calculated as \( R_{o1} = (10 \, \text{k}\Omega)/r_o \). Likewise, \( f_{b2} \) is the experimental –3-dB frequency of the circuit of Fig. 7, whose output resistance is calculated as \( R_{o2} = (3.3 \, \text{k}\Omega)/(11 \, \text{k}\Omega)/r_o \). Moreover, \( R_t \) is calculated in both cases via Eq. (11). Thus, solve Eqs. (14) to estimate \( C_\pi \) and \( C_\mu \) for this particular BJT sample and under the present collector bias conditions. Are their values typical?

**Hint:** First, take the difference \( 1/f_{b2} - 1/f_{b1} \), and solve for \( C_\mu \). Then, back substitute into Eq. (14) and solve for \( C_\pi \).

**The Cascode Configuration:**
In the common-base (CB) configuration, one side of \( C_\mu \) is at ac ground, so this configuration does not suffer from the Miller effect and is inherently faster than the CE configuration. However, it exhibits much lower input impedance than the CE configuration, and this is generally a drawback in voltage-type amplification. The cascode configuration ingeniously combines the advantages of both the CE and the CB configurations, if at the expense of an additional BJT. With reference to Fig. 8, we note that the low-frequency gain of the CE stage is only about –1 V/V, indicating that the Miller effect increase \( C_\mu \) only by a factor of 2, much less than in the basic CE amplifier of Fig. 6.

The mid-frequency gain of the cascode configuration can still be found via (10a), but with \( R_o \approx R_C \). Moreover, over the frequency range of interest, the response is governed by the dominant pole of \( Q_1 \), indicating that the upper pole frequency can be estimated as

\[
f_b \approx \frac{1}{2\pi [(R_i + r_i)/r_e] \times [C_\pi + 2C_\mu]}
\]  

(16)
9: With power off, assemble the circuit of Fig. 8. Note that the circuit is similar to that of Fig. 6, except for the insertion of $Q_2$ in series between $Q_1$ and $R_C$, and the addition of $R_3$ and $D_1$ to suitably bias $Q_2$. Then, apply power and proceed along the lines of Step MC3 to measure the mid-band gain $a_0$ as well as the upper pole-frequency $f_b$. What is the GBP of your amplifier? How does it compare with that of the CE amplifier?

**Note:** Should $f_b$ exceed the upper frequency limit of your waveform generator, estimate it via time-domain techniques, as illustrated in Fig. 2b. That is, you first measure the time constant $\tau$ governing the transient response, and then calculate $f_b = 1/(2\pi\tau)$.

**S10:** Simulate the circuit of Fig. 8 via PSpice using the 2N2222 and 1N4148 models available in PSpice’s Library. Hence, use the cursor facility of PSpice to determine $a_0$ and $f_b$. Considering that your measurements are based on two particular 2N2222 samples, while the PSpice models are based on typical 2N2222 data, comment on the degree of agreement between measurements and simulation.

**The CC Configuration:**
Like the CB amplifier, the common-collector (CC) amplifier, also called emitter follower, does not suffer from the Miller effect; so it too is an inherently fast configuration. Moreover, as shown in the version of Fig. 9, it lends itself to be driven directly from the input source with no need for any external capacitors,
indicating its ability to operate all the way down to DC.

**M11:** With power off, assemble the circuit of Fig. 9, and adjust the waveform generator for a 5-V peak-to-peak, 0-V DC offset *sine wave*. Then, apply power, and increase the input frequency until the output amplitude drops to 70.7% of its low-frequency value. Clearly, this represents the -3-dB frequency of your circuit.

*Note:* Should $f_{-3\,\text{dB}}$ exceed the upper frequency limit of your waveform generator, estimate it via *time-domain techniques*, as illustrated in Fig. 2b. That is, you first measure the time constant $\tau$ governing the transient response, and then calculate $f_b = 1/(2\pi\tau)$.

**S12:** Simulate the emitter follower of Fig. 9 via PSpice using the 2N2222 model available in PSpice’s Library. Display the *magnitude plot* of its gain $a(jf) = V_o/V_s$, and use the cursor facility to find its *dc value* as well as its $-3\,\text{-dB frequency}$.

**S13:** Using PSpice, along with a suitable test source, first at the input, then at the output, display the magnitude plots of the *input impedance* $z_i(jf)$ seen looking into the base and the output impedance $z_o(jf)$ seen looking into the emitter of the CC amplifier of Fig. 9 (use log-log scales). Hence, use the cursor facility to find their low-frequency and high-frequency *asymptotic values* as well as their *pole* and *zero* frequencies. Comment on your results. And be a happy engineer!