Guidelines of Take-home Exam
(Due to 04/20)

- In taking this exam, I (the examinee) understand that I may not work with anyone else, including conferring with others (student, or anyone else); exchanging information, answer or ideas; or in aiding or being aided by others in the completion of this assignment. I understand that failure to follow this rules is considered cheating, and may subject me to a significant reduction in my grade at the discretion of the professor. I certify that I have personally prepared the answers to this assignment in accordance with the above stated rules.

- Signature of the examinee:.................................................................
Problem-1 (10%):
Determine the peak response of the one-story industrial building of Example 1.2 to
ground motion characterized by the design spectrum of Fig.6.9.5 scaled to a peak
ground motion acceleration of 0.25g.
(a) For north-south excitation determine the lateral displacement of the roof and the
bending moments in the columns.
(b) For east-west excitation determine the lateral displacement of the roof and the
axial force in each brace.

From Example 1.2:
\[ w = 30 \times 30 \times 20 = 18,000 \text{ lbs} = 18.0 \text{ kips} \]
\[ m = \frac{w}{g} = 0.04663 \text{ kip - sec}^2/\text{in.} \]
\[ k_{N-S} = 38.58 \text{ kips/in.} \]
\[ k_{E-W} = 119.6 \text{ kips/in.} \]

(a) North-South excitation
1. Determine the natural vibration period.
\[ T_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{m}{k_{N-S}}} = 2\pi \sqrt{\frac{0.04663}{38.58}} \]
\[ T_n = 0.219 \text{ sec} \]

2. Determine the pseudo-acceleration.
From Fig. 6.9.5 scaled by 0.25, the pseudo-acceleration for \( T_e = 0.219 \text{ sec} \) is
\[ A = 0.25(2.71g) = 0.6775g \]

3. Compute peak responses.
The peak lateral displacement \( u_o \) is
\[ u_o = D = \left( \frac{T_e}{2\pi} \right)^2 A = \left( \frac{0.219}{2\pi} \right)^2 (0.6775 \times 386) \]
\[ u_o = 0.316 \text{ in.} \]

To determine bending moments in the columns, we first determine the equivalent static force:
\[ f_{se} = \frac{A}{g} w = 0.6775 \times 18.0 = 12.19 \text{ kips} \]
The bending moments in the columns are determined from the static analysis of the frame subjected to the equivalent static force, \( f_{se} = 12.19 \text{ kips} \).
Each column carries \( 1/4 \) of the force:

\[ M = \frac{f_{se}}{4} \]

\[ 2M = \frac{12.19}{4} \times 12 = \frac{12.19}{4} \times 12 \]
\[ M = 18.3 \text{ kip-ft} \]
Alternatively, from Eq. (A1.1) in the book with \( u_a = u_c, u_b = \theta_a = \theta_b = 0 \),

\[
M = \frac{6EI}{k^2} u_0 = \frac{6 \times 29000 \times 82.8}{(12 \times 12)^2} \times \frac{0.316}{12} = 18.3 \text{ kip-ft}
\]

The bending moment diagram drawn on the compression side is shown in the accompanying figure.

(b) East-West excitation

1. Determine the natural vibration period.

\[
T_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{m}{k_{E-W}}} = 2\pi \sqrt{\frac{0.04663}{119.6}} = 0.124 \text{ sec}
\]

2. Determine the pseudo-acceleration.

From Fig. 6.9.5 scaled by 0.25 the pseudo-acceleration for \( T_n = 0.124 \text{ sec} \) is

\[
A = 0.25(1.170 \times 0.124^{0.704} g) = 0.6728 g
\]

3. Compute peak responses.

The peak lateral displacement \( u_\phi \) is

\[
\begin{align*}
u_\phi &= D = \left( \frac{T_n}{2\pi} \right)^2 A = \left( \frac{0.124}{2\pi} \right)^2 (0.6728 \times 386) \\
u_\phi &= 0.1013 \text{ in.}
\end{align*}
\]

The equivalent static force is

\[
f_{S_0} = \frac{A}{g} w = 0.6728 \times 18.0 = 12.11 \text{ kips}
\]

Neglecting the lateral resistance of the columns, the axial force in each brace is

\[
P_{\text{brace}} = \frac{f_{S_0} / 4}{\cos \theta} = \frac{12.11 / 4}{0.8575} = 3.53 \text{ kips}
\]
Problem-2(5%):
Consider a vertical cantilever tower with lumped weight $W$, $T_n=2$ sec., and $f_y = 0.112W$. Assume that $\zeta = 5\%$ and elastoplastic force-deformation behavior. Determine the lateral deformation for the elastic design spectrum of Fig. 6.9.5 scaled to peak ground acceleration of $0.5g$.

For a system with $T_n = 2$ sec., Fig. 6.9.5 gives $A = (1.80g/2)0.5 = 0.45g$. Equation (7.12.1) gives

$$\frac{A_y}{g} = \frac{f_y}{w} = \frac{0.112w}{w} = 0.112$$

and Eq. (7.12.2) leads to

$$R_y = \frac{A}{A_y} = \frac{0.45g}{0.112g} = 4$$

Knowing $R_y$, $\mu$ can be computed from Eq. (7.11.2) for $T_n = 2$ sec.

$$\mu = R_y = 4$$

Then Eq. (7.12.3) gives

$$\mu_m = 4 \frac{1}{4} \left( \frac{2}{2\pi} \right)^2 0.45g = 17.6 \text{ in.}$$
Problem-3(10%):

1. Using the definition of stiffness and mass influence coefficients formulate the equation of motion for the three-story shear frames with lumped masses shown in the figure below. The beams are rigid in flexure. Neglect axial deformation.

2. Determine the natural vibration frequencies and modes; express the frequencies in terms of \( m \), \( EI \), and \( h \). Sketch the modes and identify the associated natural frequencies.

3. Comment on the effect of the column support condition on the vibration properties.

When the columns are hinged at the base, the stiffness of the first story is

\[
k_1 = 2 \left( \frac{3EI}{h^3} \right) = \frac{6EI}{h^3}
\]

The stiffness of the second and third stories does not change. Following the procedure in Problem 9.7 gives
\[ k = \frac{24EI}{h^3} \begin{bmatrix} 1.25 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \]

The mass matrix is the same as in Problem 10.12:

\[ m = \begin{bmatrix} 1 \\ 1 \\ 0.5 \end{bmatrix} \]

Then

\[ k - \omega^2 m = \frac{24EI}{h^3} \begin{bmatrix} 1.25 - \lambda & -1 & 0 \\ -1 & 2 - \lambda & -1 \\ 0 & -1 & 1 - 0.5\lambda \end{bmatrix} \] (a)

where

\[ \lambda = \frac{mh^3}{24EI} \omega^2 \]

Substituting Eq. (a) in Eq. (10.2.6) gives the frequency equation:

\[ 4\lambda^3 - 21\lambda^2 + 24\lambda - 2 = 0 \]

Following the procedure of Problem 10.12 we obtain

\[ \omega_1 = 1.4726 \sqrt{\frac{EI}{mh^3}} \quad \omega_2 = 6.0413 \sqrt{\frac{EI}{mh^3}} \quad \omega_3 = 9.3453 \sqrt{\frac{EI}{mh^3}} \] (b)

\[ \phi_1 = \begin{bmatrix} 0.8234 \\ 0.9548 \\ 1 \end{bmatrix} \quad \phi_2 = \begin{bmatrix} -0.8851 \\ 0.2396 \\ 1 \end{bmatrix} \quad \phi_3 = \begin{bmatrix} 0.3430 \\ -0.8195 \\ 1 \end{bmatrix} \] (c)

The structure with columns hinged at the base is more flexible than the structure with clamped columns, and thus has lower natural frequencies. The fundamental frequency is less than half, whereas the higher frequencies are affected less.

The modes are also affected by the column fixity. Notice that the fundamental mode of the structure with hinged columns indicates a flexible first story relative to the other stories.