Problem 1 (15%)
Assume you are the manager of two-power plant of X and Y. Each plant, at any given time, is either idle or generating electricity.
1. Define all possible outcomes
2. Assume that event A is denoted to Plant X that always in idle. Event B is denoted that at least one of the plant generating electricity. Assume that all outcomes have equal probability to occur. Find \( P(A \cap B) \) and \( P(A \cup B) \)

Solution:
1. Sample Space, \( S \) (all possible outcomes): “0” is denoted the plant, which is idle, 1 is denoted the plant, which is in generating electricity

\[ S = \{(0, 0), (0, 1), (1, 0), (1, 1)\} \]

event(A) = \{(0, 0), (0, 1)\}, \( P(A) = 0.50 \)

event(B) = \{(0, 1), (1, 0), (1, 1)\}, \( P(B) = 0.75 \)

2. \( A \cap B = \{(0, 1)\}, \ P(0, 0) = P(0, 1) = P(1, 0) = P(1, 1) = 0.25 \)

\( P(A \cap B) = 0.25, \)
\( P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1.25 - 0.25 = 1 \)

Or,
\( P(A \cup B) = 1 - P(A \cup B)' = 1 - 0 = 1 \)

Problem 2 (10%)
Find the probability of getting at least 8 heads in 8 flips of a fair coin?

Use the binomial distribution for \( p = 0.5 \), \( n = 8 \) and \( r = 8 \)

\[ P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}, \] \( \binom{n}{r} = \frac{n!}{r!(n-r)!} \)

\[ P(X = 8) = \binom{8}{8} 0.5^8 (1 - 0.5)^{8-8} = 1 * 0.5^8 * 1 = 0.0039 = 0.39\% \]
Problem 3 (10%)
There is a constant probability of p=0.25 that milk container is underweighted in milk packaging facility. At the batch 20-milk container are selected. Manager claims that there is 10% chance that exactly 3-milk container would be found as underweighted. Is he right?

Use the binomial distribution for p=0.25 , n=20 and r=3

\[
P(X = r) = \binom{n}{r} p^r (1 - p)^{n-r}
\]

\[
P(X = 3) = \binom{20}{3} 0.25^3 (1 - 0.25)^{20-3} = \frac{20!}{3!(20-3)!} \times 0.25^3 \times 0.75^{17}
\]

\[
= 0.1339 = 13.4% > 10%
\]
No. Manager is not right.

Problem 4 (20%)
An printed circuit board manufacture produces 1000 boards per day having a mean impedance value of 100 ohms. QA tests indicate 80% of the boards produced have impedance between 95 and 105 ohms. If the minimum acceptable impedance range is from 90 ohms to 110 ohms, how many boards are rejected per day? Assume a normal distribution of impedances.

Population follows normal distribution. So; \( \overline{x} \approx \mu = 100 \) Area=0.40 use z-table;

\[
z = \frac{x - \mu}{\sigma} = \frac{-5}{3.906} = -1.28 \quad \sigma = 3.906\text{ohms}
\]

for acceptable impedance range:

\[
z = \frac{x - \mu}{\sigma} = \frac{-10.0}{3.906} = -2.56
\]

use , z table; Area: 0.4948 2* 0.4948=98.96% accepted rejection criterion: \( 1 - 2 \times P(0 \leq z \leq 2.56) = 1.04% \) boards rejected: 1000*0.0104= 10
Problem 5 (15%)
The results if the EIT exam had indicated a mean score of 68 and standard deviation of 14. Assuming a normal distribution, what percent of the people scored above 95?

\[ z = \frac{x - \mu}{\sigma} = \frac{95 - 68}{14} = 1.93 \]

Area from z table : 0.4732  
\[ P(X \geq 95) = 0.5 - P(0 \leq z \leq 1.93) = 0.5 - 0.4732 = 0.0268 = 2.68\% \]

Problem 6 (15%)
Ten steel rods were selected at random from output of a milling machine that produces 1000 rods per day. The diameter of the rods was measured with the results shown in the following.

Mean diameter, \( \bar{x} = 0.516 \), sample size, \( n = 10 \)  
Sample standard deviation, \( s = 0.0392 \)

What interval of diameters falls within a 95% confidence level?

\[ \mu = \bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}, \quad 1 - \alpha = 0.95, \quad (\alpha / 2) = 0.25, \quad v = n - 1 = 9 \]

use t-table: \( t_{\alpha/2} = 2.262 \),  
\[ \mu = 0.516 \pm 2.262 \frac{0.0392}{\sqrt{10}} = 0.516 \pm 0.028 \]
Problem 7 (15%)
In a certain circuit design, it is important that the standard deviation of the current be less than 20 mA. In a test on 13 parts, the sample standard deviation found to be 12 mA. Can we 99% confident that standard deviation will not exceed 20 mA.

\[
\frac{(n-1)S^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{(1-\alpha/2)}}
\]

\(1 - \alpha = 0.99\) \hspace{1cm} \(v = n - 1 = 13 - 1 = 12\),
\((\alpha / 2) = 0.005\) \hspace{1cm} \text{use Chi-squared table: } \chi^2_{0.005} = 28.3
\((1 - \alpha / 2) = 0.995\) \hspace{1cm} \text{use Chi-squared table: } \chi^2_{0.995} = 3.0738

\[
\frac{(13-1)12^2}{28.3} \leq \sigma^2 \leq \frac{(13-1)12^2}{3.0738}
\]

61.06 \leq \sigma^2 \leq 562.2

7.81 \leq \sigma \leq 23.7

No. Standard deviation of the current of the parts exceed 20 mA with the confidence level of 99\%. 