Prob-1 (2.5%) A particle moves along the $x$-axis with an initial velocity $v_x = 18 \text{ m/sec}$ at the origin when $t = 0$. For the first 4 seconds it has no acceleration, and thereafter it is acted on by a retarding force which gives it a constant acceleration $a_x = -4 \text{ m/sec}^2$. Calculate the velocity and the $x$-coordinate of the particle for the conditions of $t = 8 \text{ sec}$ and $t = 12 \text{ sec}$ and find the maximum positive $x$-coordinate reached by the particle.

\[
\begin{align*}
\text{Prob-2 (2.5%)} & \quad \text{The curvilinear motion of a particle is defined by } v_x &= 50 - 16t \text{ and } y = 100 - 4t^2, \text{ where } v_x \text{ is in meters per second, } y \text{ is in meters, and } t \text{ is in seconds. It is also known that } x = 0 \text{ when } t = 0. \text{ Determine its velocity and acceleration when the position } y = 0 \text{ is reached.}
\end{align*}
\]
Prob-3 (2.5%) To anticipate the dip and hump in the road, the driver of a car applies her brakes to produce a uniform deceleration at A. The speed of car is 110 km/h at the bottom A of the dip. If the total acceleration at A is 4 m/s² and acceleration vector 145° respect to horizontal direction and the radius of curvature of the hump at C is 150 m, calculate (a) the radius of curvature at A, (b) the acceleration at the inflection point B, and (c) the total acceleration at C.

\[
\begin{align*}
\alpha &= -\alpha_A \cos 35° = -3.28 \text{ m/s}^2 \\
\alpha_n &= \sqrt{\alpha_A^2 - \alpha_t^2} = \sqrt{4 - 3.28} = 2.29 \text{ m/s}^2 \\
\rho &= \frac{\alpha_A}{\alpha_n} = \frac{4}{2.29} = 407 \text{ m} \\
\end{align*}
\]

Prob-4 (2.5%) Car A is accelerating in the direction of its motion at the rate of 2 m/sec². Car B is rounding a curve of 150 m radius at a constant speed of 45 km/hr. Determine the velocity and acceleration which car B appears to have to an observer in car A if car A has reached a speed of 70 km/hr for the positions represented.
Prob-5 (2.5%) Pin C is attached to rod BC and slides freely in the slot of rod OA which rotates at the constant rate $\omega=1.0 \text{ rad/s}$. At the instant $\beta=30^\circ$ determine angular velocity and angular acceleration of rod OA. Show velocity and acceleration components and directions on the figure below.

$$r = \left(1 + \sin 60^\circ\right)^2 = 1.93\text{ m}$$

$$v = \omega (r) = 1.93\sqrt{3}$$

$$\alpha = \frac{v^2}{r} = \frac{12}{1.93} = 6.18\text{ rad/s}^2$$

$$v_{\theta} = v \cos 15 = r \dot{\theta}, \quad \dot{\theta} = 6.18\text{ rad/s}$$

$$v_{r} = -v \sin 15 = -r, \quad \ddot{r} = 0.26$$

$$\ddot{\theta} = \frac{r \ddot{r} + 2 \dot{r} \dot{\theta}}{r} = -0.26 + \frac{2 \cdot 0.26 \cdot 6.18}{1.93} = 0$$

Prob-6 (6 to 15, 1% at each)

Given: $r(t)$ is a position vector of a point in a rectangular coordinate system where:

$$x = 3t^2 \quad \rightarrow \quad r = \left<3t^2, 4t\right>$$

$$\dot{r} = \left<6t, 4\right>$$

What is the magnitude of the acceleration and velocity vectors at $r(2)$?

Solution:

$$a_x = 12, \quad a_y = 4$$

$$\ddot{x} = 6, \quad \ddot{y} = 0$$

$$v_x = 12, \quad v_y = 4$$

$$\ddot{r} = \left<12, 4\right>$$

$$v = \left<12, 4\right>$$

Prob-7 Given: $r$ is a position vector of a point in a polar coordinate system as shown below where:

$r = 2 \phi, \quad \phi$ is in meters

$\phi = t^2, \quad t$ is in radians, (t) is in seconds

What is the velocity of the point when $r$ is at 60 degrees?

$$r = 2t^2$$

$$v = \left<v_r, v_\theta\right>, \left<\dot{r}, \dot{\theta}\right>$$

$$\dot{r} = 4t \quad \dot{\theta} = 2t$$

$$\theta = 60^\circ = \frac{\pi}{3} \text{ rad}$$

$$t = \sqrt{\frac{\theta}{18^\circ}} = \sqrt{\frac{\pi}{3}}$$

$$r = 4\sqrt{\frac{\pi}{3}}$$

$$v = 2\frac{\pi}{3} \left(2\sqrt{\frac{\pi}{3}}\right) = \frac{M}{4}$$

$$\therefore$$

$$v = \phi^\prime = \sqrt{\phi^2 + \left(\frac{\phi^\prime}{\phi}\right)^2} = \sqrt{16\frac{\pi^2}{3} + 16\frac{\pi^2}{3}} = 4.78$$
Prob-8 Given: \( \mathbf{r} \) is a position vector of a point in a polar coordinate system as shown below where:
\( r = 2 \), \( \phi \) is in radians, \( t \) is in seconds

The acceleration of the point at \( t = 2 \) seconds is:

\[
\begin{align*}
\mathbf{a} &= \mathbf{a}_t \quad \mathbf{a}_\phi \\
\mathbf{a}_t &= \mathbf{a} - \mathbf{a}_\phi \\
\mathbf{a}_\phi &= \dot{\mathbf{a}} + 2 \dot{\mathbf{r}} \dot{\phi}
\end{align*}
\]

\[
\begin{align*}
r &= 2t^2 & \ddot{r} &= 4t \\
\dot{\phi} &= \frac{4}{r} & \ddot{\phi} &= -1 \\
\mathbf{a}_t &= 4 - 8(16) = -128 \\
\mathbf{a}_\phi &= (8)(2) + (2)(8)(4) = 16 + 64 = 80 \\
\end{align*}
\]

\[
\mathbf{a} = \sqrt{a_t^2 + a_\phi^2} = \sqrt{(-128)^2 + 80^2} = 197.6 \text{ m/s}^2
\]

Solution:

Prob-9 What is the tangential velocity \( (v_t) \) of the particle shown below given \( v_r \) (radial velocity) = 10 and \( v_\phi \) (transverse velocity) = 5.777 (Note, angle between \( v_r \) and \( v_\phi \) = 30 degrees)

\[
\begin{align*}
v_t &= \sqrt{v_r^2 + v_\phi^2} \\
&= \sqrt{10^2 + 5.777^2} = 11.54 \text{ m/s}
\end{align*}
\]

Solution:

Prob-10 A 4 meter long ladder as shown below is sliding down the wall. If Point B is moving at 2 meters per second, what is the velocity of Point A?

\[
\begin{align*}
x^2 + y^2 &= 16 \\
2x \ddot{x} + 2y \ddot{y} &= 0 \\
x \ddot{x} + y \ddot{y} &= 0
\end{align*}
\]

\[
\begin{align*}
\ddot{x} &= -\frac{2}{y} \\
&= -\frac{2}{4} = -\frac{1}{2} \\
\ddot{y} &= -\frac{6}{\sqrt{3}/3} = -\frac{6}{\sqrt{3}} = -3.46 \text{ m/s}
\end{align*}
\]

Solution:

Prob-11 A particle moves in a circular path as shown. If \( r = 50 \) meters and if the distance (S, arc length in meters) the particle travels from its initial position is defined as \( S = 6t^2 + 2t \). What is the particle's angular acceleration and velocity at \( t = 2 \) sec?

\[
\begin{align*}
\mathbf{a} &= \frac{dS}{dt} = 12t + 2 \\
\mathbf{v} &= r \dot{\theta} \quad \dot{\theta} = \frac{22}{50} = 0.44 \text{ rad/s} \\
\mathbf{a}_t &= \ddot{r} = 12 = r \ddot{\theta} \quad \ddot{\theta} = \frac{12}{50} = 0.24 \text{ rad/s}^2
\end{align*}
\]

Solution:
Prob-12 A particle moves along a straight line according to the equation $s = 3t^2 - 5t$. What is the particle's velocity when $t = 10$ sec.?

Solution:

$$u = \frac{ds}{dt} = 6t - 5 = 60 - 5 = 55 \text{ m/s}$$

Prob-13 A 1" diameter steel ball bearing is dropped from the top of the Sears Tower in Chicago. If the steel ball falls 1000 ft, what is its speed just before it hits the pavement below? (Neglect air resistance).

Solution:

$$h = 1000 \text{ ft}$$

$$u^2 = 2gh + 2\frac{g}{2}h = 2\left(\frac{g}{2}\right)(1000) = 253.8 \text{ ft/s}$$

Prob-14 A ball is thrown with an initial velocity of $(v)$. At what angle from the ground should the ball be thrown in order to maximize the horizontal distance the ball will travel? Neglect air resistance.

Solution:

$$\theta = 45^\circ$$

Prob-15 A stone is thrown off of a 100 m cliff at 0 degrees with respect to the ground at an initial velocity of 120 km/hr. shown below. How far will the stone travel horizontally before it hits the ground below? Neglect air resistance.

Solution:

$$x = vt$$

$$h = \frac{1}{2} gh^2$$

$$\frac{h}{x^2} = \frac{g}{2v^2}$$

$$x = \frac{2h}{v^2}$$

$$x = \sqrt{\frac{2(100)(120)}{9.8(3.6)}} = 150.5 \text{ m}$$