Knowledge for Teaching Mathematics:

What do teachers know? What should teachers know?

In the field of elementary education, people tend to assume that if someone can do math, then they can teach math. I learned long division in fourth grade, and I have not forgotten how to do it, therefore I should be able to effectively transfer knowledge to a fourth grader in my classroom. This assumption is not always accurate, as it turns out that there are more components to understanding mathematics than simply being able to solve certain problems. The result is that teachers with an incomplete understanding of the subject affect the ways their students understand and feel about mathematics. These students also end up with an incomplete understanding, and more often than not, also have negative feelings about math. There have been several studies exploring the relationship between what elementary school teachers know and what they should know when it comes to teaching math. Many of these studies assert that there is a gap between actual knowledge and necessary knowledge. Teachers who have accurate and comprehensive mathematical knowledge are typically more successful at producing students with similar knowledge.

Deborah Lowenberg-Ball, the Dean of the School of Education at the University of Michigan, Ann Arbor, pioneered a study in which she interviewed and surveyed 252 prospective teachers who were majoring in either elementary education or mathematics. She collected information about their subject-matter knowledge as well as about their
teaching capabilities. Her conclusions disprove the assumptions that elementary math concepts are easy, that prospective teachers learn much of the content they need to know before college, and that majoring in math provides a complete understanding of mathematics (Ball 1990). If Ball is correct in saying that elementary school math is difficult, that teachers do not begin college with enough content knowledge, and that math majors do not always have a complete understanding of mathematics, then what is adequate understanding for teaching mathematics?

Ball did another study with two of her researchers, Mark Thames and Geoffrey Phelps, about the importance of content knowledge in teaching mathematics. They found that teachers with a high level of pedagogical knowledge are better equipped to teach mathematics, and their students demonstrate higher levels of achievement. Pedagogical knowledge, according to this study, is having access to multiple examples, methods, explanations, illustrations, and demonstrations of a concept, and being able to choose the most effective methods in order to make the concept explicit to others (Ball 2008). This points out that, though there are many ways to correctly explain mathematical concepts, some are more useful in teaching math. Effective teachers with a firm grasp of their subject, as well as an understanding of the most appropriate way to teach it, will have better results.

Christine Perry defines a few important learning goals for understanding this issue. Performance-based learning means that the desired result of learning is a satisfactory test score or grade, whereas mastery is learning for the sake of learning, with knowledge as the goal (Perry 2011). Mastery is a more effective goal, as it encourages complete understanding of a subject and the motivation lies within the student, rather than coming
from an extrinsic reward or punishment. Mastery is not determined by a student’s successful completion of a skills test, such as a minute drill, but must be tested more thoroughly. If an individual can complete a math problem, they should also be able to identify why it makes sense, and when one might encounter such a problem in real life. Testing for mastery involves making sure that a student has a very complete knowledge of a problem, not just the ability to solve it.

Perry’s study finds that teachers who personally strive for mastery in mathematics are typically more effective math teachers, but that there is often a disconnect between teachers’ personal goals and the goals they set for their students. A typical math classroom in the United States encourages performance-based learning, regardless of the teacher’s intentions. This is likely due to increased focus on testing as a means of assessment, and also stems from the fact that it is difficult for teachers to ignore their own experiences when learning how to teach. Perry states that a university education about appropriate methods for teaching mathematics often cannot compete with a teacher’s memories of how they were taught in elementary school (2011). Moving classrooms to adopt mastery as their goal will improve success rates in mathematics learning. Teachers who adopt mastery as their personal goal will spend more time making sure that they really understand what they are teaching, and this will have positive effects on the abilities and attitudes of their students.

Ball also asserts the value of mastery to mathematics education. She criticizes standard math curriculum for being rule-bound and focused on procedure, rather than on the relationship between concepts. Knowing how to do math problems only requires the ability to follow directions, and those who can solve a long division problem are
demonstrating little more than their ability to memorize steps and complete them in order. There is no requirement that they understand what specifically is happening when division occurs. This type of understanding is not useful to students, because they do not know when to divide something, and why it works. Using this model, each new procedure is introduced as a new topic, rather than showing how mathematical concepts are all related (Ball 1990). You use a different procedure to multiply fractions than you do to multiply whole numbers, but the reason you are multiplying is the same. Many teachers lack conceptual knowledge about what is actually happening to a number when it is multiplied, divided, etc. and those teachers will not be able to pass a complete understanding on to their students.

College level coursework that revisits elementary mathematical concepts can be helpful to pre-service teachers in order to remind them what they need to know, and ensure that they are headed toward mastery of mathematics. Even though these future teachers may not have had ideal math education in elementary school, it is not too late for them to learn the underlying concepts that make basic mathematical procedures possible. Successful courses, such as Math 165 and Math 565 at San Francisco State University, would not necessarily focus on teaching methodology, but rather on how to understand concepts and articulate those concepts using multiple explanations.

According to Ball, mastery in mathematics education involves being able to do a problem, explain why it works, and come up with an example of when it would be used (1990). Her research found that most prospective teachers were rarely able to come up with strong examples or scenarios for more complex concepts, such as dividing with fractions. Even the teachers who were able to correctly develop a story problem were not
able to assign a natural or intuitive meaning. For example, one man said that, to illustrate dividing fractions, he would slice a pizza and then stack $\frac{1}{2}$ of one pizza on top of $1 \frac{3}{4}$ of another pizza and see how many times $\frac{1}{2}$ fits into $1 \frac{3}{4}$. A student’s response to this type of problem would likely question the point of stacking pizzas. In order to effectively teach math, a teacher needs to be comfortable enough with the subject in order to explain it in multiple ways, and to be able to assess the validity of a method of explanation they had not considered before. They need to be able to guide students in solving problems by reframing questions, giving examples, and providing hints that will make concepts clear. In terms of subject matter knowledge for teachers, Ball summarizes the requirements as follows: Teachers should be able to correctly perform mathematical procedures, understand the underlying principles, and understand how different mathematical concepts are related to one another (1990).

A study by Rae Young Kim, et al. compares teacher education programs in the United States with those in South Korea. Through comparing practices in these two countries, they have determined that teaching mathematics involves knowledge in multiple areas, including “content knowledge, pedagogical content knowledge, general pedagogical knowledge, curricular knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of educational ends, purposes and values and their philosophical grounds” (2011). In other words, teachers not only need to understand how to do math, they need to understand why it makes sense, how to teach it, when to teach it, how their students will react to it, why it is important to learn, etc. This is no longer a simple question of knowing how to do long division and then telling a child how to do it. According to Jasmine Ma and Marcy Singer-Gabella, it is necessary, though often
difficult, for prospective teachers to understand mathematics from multiple directions (2011). Simply being familiar with concepts is not enough. They must understand what it is like to know mathematics, to learn it, and to teach it. Teacher training classrooms typically focus on one of these. Some classes emphasize mastery of mathematical concepts, while others emphasize mastery of teaching methods, but both are essential.

What do teachers need to know about mathematics in order to teach, and how does this differ from what they commonly know? These studies have shown that a complete understanding of mathematics involves not only the ability to solve problems, but also awareness of why these methods make sense, and why they are employed. Effective math teaching requires mastery by the teacher, as well as the goal of mastery for the students. Finally, in addition to adequate subject matter knowledge, teachers must also have adequate knowledge of teaching methods, the importance of the subject, and the context in which their students are learning. Teachers commonly feel unprepared to teach mathematics, as a result of incomplete understanding stemming from the value placed on a performance-based curriculum.
References


