Chapter Four: Social Dilemma

4.1 Introduction

Competitive behavior has often been presented as the ultimate engine of economic, technological, and scientific progress. Since all such progress is likely to imply some measure of social benefit, competition has been widely praised in modern western culture. However, competition can have very negative effects on social well-being. An early example is known as the "tragedy of the commons." The commons used to be playgrounds enjoyed by small town communities in seventeenth century England. But when some villagers brought their sheep to graze on the commons a dilemma arose: those who did could raise more sheep and make more money. But if all villagers did, overgrazing would destroy the commons to the detriment of all.

A. W. Tucker is credited with the following version of the problem. It is known as the Prisoner's Dilemma and goes as follows: Two suspects are taken into custody and separated. The district attorney is certain that they are guilty of a specific crime, but he does not have adequate evidence to convict them at a trial. He presents each of them with the following bargain: each suspect may either confess or remain silent, knowing that his presumed accomplice has the same two options. If both confess, they will be prosecuted, but not for the most severe sentence. If neither confesses, they will be prosecuted and probably convicted on a minor charge. But if one confesses while the other does not, the confessor will get very lenient treatment while the other will get the harshest possible one. Uncertain about his accomplice's intent, each suspect may find it best to confess. This results in a serious sentence rather than a minor one, should they instead trust each other. This scheme is the basis of the widespread practice of plea bargains in the American judiciary. Whenever several accomplices are involved in a suspected crime and evidence is lacking, the offer of lenient treatment in return for cooperation with the prosecution often results in the confession of at least one suspect.

The same structure is found in other areas of social interaction. Economic competition is often seen as the best way to ensure the best quality and lowest prices for the public. This may be true only up to a point. Excessive competition may result in price wars, lead producers to sell below costs and eventually go bankrupt. It is not always clear that such "shakeouts" result in better quality and prices for the public, not to mention the loss of jobs and revenues this may imply. However, the lessening of competition due to the formation of cartels or to the practice of "price fixing" can have very negative effects. Contrary to the problem of the commons or to the practice of plea bargains, cooperation here could result in excessive prices or lower quality and is highly undesirable.

International relations are another area where such dilemmas abound. Arms rivalries are perfect examples. Two nations may enjoy the security and economic benefits
of balanced and low arsenals. But if one side decides to increase its arms stocks, the two may get caught in a dilemma: not arming results in a dangerous loss of security while arming may only spur further buildup with dire economic consequences and little improvement in security. Worse, history shows that arms races often lead to war. Unfortunately, spontaneous arms control and disarmament is rare. It is often necessary to engineer arms control and disarmament schemes in order to stabilize and control arms rivalries. Trade disputes provide another major example of dilemma. If two nations erect trade barriers against each other, they both suffer from the impediment to free trade. If they both lower their barriers, they enjoy the benefits of the exchange of goods under the sole influence of market forces. If one side defects by erecting barriers, it enjoys free access to the other's market while protecting its own industries.

4.2 Simple Game Models of Social Dilemma

The most basic game model of the Prisoner's Dilemma is illustrated in Table 4.01. Each side has the two choices (and strategies) "Confess" and "Not Confess." In more cases, the choices will be called "Defect" and "Cooperate." The values of the payoffs are typical. One easily sees that Prisoner 1, for instance, is better off choosing Defect over Cooperate regardless of what Prisoner 2 does. In the language of game theory, one says that Defect dominates Cooperate in this game. Since the situation is symmetric, the rational outcome of a one-shot play in the absence of any enforceable agreement between the two sides is (Defect, Defect) which results in a worse outcome than the equitable and more rewarding (Cooperate, Cooperate).

![Table 4.01: The Prisoner's Dilemma](image)

This game can be easily generalized to more than two players. Table 2 shows what a three way Prisoner's Dilemma looks like. The front table describes the payoffs when Player 3 decides to cooperate (not confess) and the back table describes them when he defects. This structure is typical of social dilemma. Each individual is always better off defecting but the payoffs of all decrease with each additional defector (except for that defector).
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Table 4.02: A Three-Player Prisoner's Dilemma

One wonders how cooperation can develop within such a framework unless one introduces outside mechanisms such as enforceable agreements. The fact is that there are numerous examples where it indeed does. Axelrod recounts how German and French soldiers learnt to develop tacit cooperation schemes during WWI. Artillery attacks, for instance, would occur at specific and therefore predictable times and would be targeted in a way that would avoid substantial damage. The reciprocity that developed afforded both sides in that tacit bargain better chance to survive the onslaught. The key to developing such reciprocity is the expectation of a tomorrow. The decision maker who is concerned about future consequences of present decisions may choose what could appear today as an inferior course of action in order to reap higher benefits in the long run.

4.3 The Shadow of the Future

In order to represent players' concern for tomorrow, it is first necessary to assume that the game will indeed be played again. But what if it is to be played just once more or even, say, twenty seven times in all? In that case, each side might consider what should happen at the very last turn: expecting no further tomorrow, today's (the last day of the game) rational decision in a Prisoner's Dilemma is to defect. This being so, the decision on the next to last day does not affect and is therefore not affected by the expected outcome on the last day. So, rational players will also defect on the next to last day and so on, in a typical backward induction argument, back to the very first turn. If a Prisoner's Dilemma is repeated finitely many times and the players know how many times in advance, the rational outcome is a perpetual defection.

To break out from that logic, there must be either uncertainty in the number of turns or a potential infinity of future turns. As we saw in our earlier discussion of discounting these two options are in fact closely related. One can introduce a probability (01) that the game will be repeated at least once more after each turn. Now a familiar stochastic game representation of the repeated game is shown in Figures 4.01 and distinguishes two states of the repeated game. "co-co" means that that both players cooperated at the last turn while "df" means that one or both defected.
Before we solve the game of Figure 4.01, we can observe that in two instances there are two different moves from one node to another: from C12 to df, Player 2 can choose dfct or coop, and from C42 to df, Player 2 (again) can choose dfct or coop. GamePlan 2.0 does not allow such a feature. But there is a simple remedy. One simply introduces a chance node that does not change anything to the logic as in Figure 4.02.

Figure 4.02: An Equivalent Structure

Figure 4.03 is a more detailed stochastic game representation of the same repeated prisoner's dilemma that distinguishes four states instead of two. In this case, decisions will be based bot just on whether at least one player defected but exactly who did what at the last turn.
Figure 4.03: A Repeated Prisoner's Dilemma with Four States

In this repeated game approach, the players' objectives become a "discounted stream" of future outcomes. But even with this implied concern for the future, it is not yet clear that either side will see that cooperation could be a better strategy. In fact, defection is still a perfectly valid decision. Suppose indeed that Player 1 expects Player 2 to defect indefinitely. Then, Player 1 cannot do better than also defecting forever. To see this, just consider a one-turn decision to cooperate by Player 1. Since this has no effect on Player 2, Player 1 will simply lose some payoff at that turn. In fact we already know that a Nash equilibrium of the one-shot game becomes an equilibrium (Markov perfect here) of the corresponding discounted repeated game.

If however, Player 1 expects a more cooperative attitude on his opponent's part, things can change drastically. Suppose instead that Player 2 is expected to adopt the following strategy called Grim: Player 2 will begin by cooperating and will continue doing so as long as cooperation is maintained. But Player 2 will revert to permanent defection after the first defection. This is illustrated in Figure 4.04 below.

If Player 1 always cooperates, his outcome at each future turn is 0 and this results in an expected payoff (at the blue node co-co) that is also also 0. But if Player 1 unilaterally defects at any point in time, thus reaching the red node C12, he expects Player 2 to permanently revert to defection. He also knows that he will then defect forever, which will bring him the permanent outcome of (1) for all future turns and the resulting expected payoff of ( appearing under the blue node df-df. So, the benefit of first defecting () implies a discounted expected payoff for a total of . This is clearly far worse that cooperating. Moreover, the expectation of perpetual defection once either side defects initially is fully rational as can be checked by inspecting the expected payoffs at
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each node in Figure 4.02. This depicts a Markov perfect equilibrium and therefore a completely rational plan for both sides.

Figure 4.04: The Grim Strategy

The grim strategy is a well-known solution to the repeated prisoner's dilemma that results in rational cooperation. It is however a bit extreme in that it is completely unforgiving. But there are alternatives that are just as rational and somewhat more forgiving. One prominent example can be first described in words. Suppose that the two players agree to reciprocate cooperation just as in Grim. And if either (or both) side defects, they agree to jointly defect on the next turn and implement punishment. However, they do not envision remaining in punishment forever. Instead, they agree to remain in that state for a fixed, agreed upon, number of times before jointly returning to cooperation. If the time spent in punishment is sufficiently long given the discount factor, cooperating will still appear as the best option while in cooperation. This is a bit complicated to picture as a stochastic game as in Figure 4.01 since one has to distinguish all punishment periods thus creating a very large game. But there is an almost equivalent alternative that is easy to picture. Suppose that once in punishment mode the two players agree to commonly observe a random event such as the toss of a coin or the throw of a dice. If the random outcome meets a certain specification (heads or a six, for instance), punishment mode will continue. If not, the two sides will jointly return to cooperation. Figure 4.05 illustrates the case of a coin toss.

In fact, the probability of staying in punishment state determines an expected number of turns being in that state before returning to cooperation. The formula for that number of turns is 1. For the above this means spending an expected two turns in punishment mode anytime defection from cooperation occurs. The deterrent effect is clearly enough as Figure 4.06 indicates.
It turns out that the toss of a coin and the resulting expected two turns of punishment is in fact too much. It can be shown analytically that a probability of remaining in punishment with an expected turns in punishment mode is deterrent enough and the minimum possible (see homework). Of course, these figures depend on the discount factor and the payoff parameters.

4.4 Tit for Tat

The Grim strategy and its more forgiving relatives have one troubling attribute in common: the victim of a unilateral defection is treated in the same way as the
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perpetrator since the punishment round is common to the two sides. The victim in fact never recoups any of his initial losses. One would wish that there be some redress for the victim involved in that punishment process. Tit-for-Tat, a strategy inspired from the biblical "an eye for an eye" is precisely such a candidate for redress. Its formulation is elegantly simple: "begin by cooperating, then do unto your opponent what your opponent did unto you." In practical terms, cooperate when your opponent just did, defect otherwise.

Clearly, two players adopting the Tit-for-Tat strategy end up cooperating forever. However, this is not enough of an argument for the game theorist. We still need to ensure that this is an optimal plan for both. A look at Figure 4.07 will clarify the issues.

Figure 4.07: Tit-for-Tat

In the upper left part of the figure that begins with the blue node "co-co", the choice to cooperate appears entirely legitimate. For Player 1, defecting yields an expected payoff of at Node C12 instead of at Node C11. And for Player 2 at Node C11, cooperating leads to "co-co" with an expected payoff of while defecting leads to "co-df" with an expected payoff So cooperation there is indeed best. In the lower right corner, the situation is a bit similar to that of the Grim strategy. Expecting defection it is best to defect. Therefore, it appears that the threat of retaliation implicit in Tit-for-Tat is enough to sustain cooperation.

But there is trouble in the other two parts of the game. At Node "co-df" for instance, the decision for Player 1 to cooperate yields an expected payoff of at Node C1 while that to defect yields an expectation of at Node C22. So, how could it be that Player 1 prefers defect to cooperate at Node "co-df". The reality is that he doesn't, and that is because Tit-for-Tat is a Nash equilibrium of the repeated game, not a Markov perfect
And since Node "co-df" is off the equilibrium path, any choice there is allowed by the Nash concept. Indeed, starting anywhere in the upper right or lower left parts of Figure 4.07, the two sides get trapped into an alternating sequence with one side cooperating while the other defects, and conversely. But this is precisely the scenario that is not optimal and that undermines the credibility of Tit-for-Tat.

This was not the case for Grim and its more forgiving relatives: expecting defection after a breach of cooperation, it is indeed optimal for both players to defect, forever or with some specified mechanism to return to cooperation. Such schemes form Markov perfect equilibria of the stochastic game that translate into subgame perfect equilibria of the repeated game.

4.5 Generosity

There are alternatives to Tit-for-Tat in seeking redress for the victim in a repeated prisoner's dilemma. One is to examine punishments based on probabilistic moves. The example shown in Figure 4.08 is typical and goes as follows: whenever their opponent cooperates, each side cooperates in return. But if one defects, then the opponent defects with probability 0.555556.

The result has several desirable properties: First, unlike Tit-for-Tat it forms a Markov perfect equilibrium and therefore ensures credibility of the probabilistic threat of retaliation. Second, just like the limited-time punishment relatives of Grim, the players would always eventually return to cooperative loop after any defection and punishment. And third, it has some of the redress flavor that we sought in Tit-for-Tat. Indeed, suppose
that Player 1 unilaterally defects from the cooperative loop between the nodes "co-co" and "C11." This moves the players to Node "df-co" in the lower left corner. But there, contrary to what Grim and its more forgiving relatives specified, the situation is tilted to the advantage of the victim (Player 2) since Player 1 is expected to cooperate fully while Player 2 may defect with probability \( p \). Of course, such a defection would lead to node "co-df" where Player 2 would be expected to cooperate fully while Player 1 (the initial perpetrator) would now be entitled to some retaliation. But the initial asymmetry between the two responses indeed favors the victim.

It turns out that the generosity implicit in the significant probability or forgiving a unilateral defection prevents the players to be trapped into the alternating sequence of defect-cooperate that characterized Tit-for-Tat and compromised the credibility of the retaliation threat. There are other Markov perfect equilibria based on probabilistic moves in this game that also support cooperation (see homework).

### 4.6 Guilt and Contrition

The developments of the previous sections suggest that the way states are defined in a stochastic game representation may have consequences on the emergence of cooperation, on its dynamic stability, and on the potential for redress. In fact, the way states are defined is closely related to the way the players see and interpret the developments of the game. Conceptually, one might want to distinguish between various types of defections. Indeed, one might defect out of temptation to exploit a cooperative opponent, but one might also defect in retaliation to the exploitative defection of one's opponent. The two types of defections should be distinguished. This suggests introducing the concept of guilt in the very definition of the states that are distinguished in the stochastic game representation. We might say that is guilty if he defects against a non-guilty opponent but is not guilty if he defects in retaliation. We might also assume that guilt can be erased once appropriately punished. We would then represent four possible states of the game according to who is guilty or not. This is done in Figure 4.09.
Figure 4.09: The Guilt Game

At Node "NN" neither player is guilty while they both are at Node "GG". At Node "NG" Player 1 is guilty but Player 2 is not and conversely at Node "GN." The game structure differs from that of Figure 4.03 in order to reflect the conventional wisdom of guilt. If at "NN" player 1 chooses to defect while Player 2 simultaneously cooperates, the game moves to Node "GN." There, Player 2 is entitled to defect rightfully while Player 1 should cooperate in order to erase his guilt. Of course, should Player 2 decide to generously cooperate, that does not perpetuate Player 1's guilt unless he failed again to cooperate. The move structure reflects what is often called the transition conditions that define how moves modify the current state of a game into the next.

Viewed through this prism of guilt, the game can be solved in various modes. In pure-perfect mode, for instance, one finds three distinct solutions. One is usually called Contrite Tit-for-Tat because it is akin to Tit-for-Tat except that contrition is factored in to avoid the alternating loop of defect-cooperate that doomed the original Tit-for-Tat. Contrite is picture in Figure 4.10.

Figure 4.10: Contrite Tit-for-Tat

It is interesting to note that this is a strict equilibrium where each move with probability one is strictly better than its alternative given all others as expectation. Contrite Tit-for-Tat is a very solid solution of the repeated prisoner's dilemma although it is surprisingly very rarely discussed in the literature. The game has two other pure-perfect solutions: One has both players choosing to always defect and thereby remain always guilty. The other exhibits a strange alternating behavior, cooperating when they are guilty and defecting when they are not. There also probabilistic solutions, some of which exhibit some generosity (see homework).
4.8 The Free-Rider Problem

A three-player prisoner's dilemma is the simplest example of social dilemma with more than two players. There the issue is not just to deter uncooperative behavior by one side. It is also to engineer some global cooperation between several parties. The problem is difficult and has many facets that cannot be treated in this space. A simple stochastic game version of the dilemma of Table 4.02 is pictured in Figure 4.11. It distinguishes only two states of the game:

[Diagram of a Three-Player Repeated Dilemma]

The chance nodes are used to simplify the presentation and potential modifications of the game. In particular, one doesn't have to worry about missing some discount factors since they are accounted for on the chance moves. Likewise, the payoffs are specified on those chance moves. Not surprisingly this game has a Grim (Markov perfect) equilibrium whereby cooperation is maintained by the threat of perpetual punishment. Evidently, there are other solutions but they require a substantial complexification of the game structure to identify and this taxes the capabilities of GamePlan to its limits. One simple modification is to introduce a probabilistic return from punishment mode as we did in the two-player case (see homework).

Homework

1. Modify the game of Figure 4.05 by increasing the probability of returning to cooperation to 0.85 and 0.9 successively (and lowering that of continuing in punishment
accordingly). Solve in pure-perfect mode, observe, and comment. What happens when is set to?  

2. Solve the game of Figure 4.03 in pure-perfect mode. Discuss what you obtain.  

3. Solve the game of Figure 4.03 in explore-perfect mode. Discuss and classify solutions according to whether they promote cooperation and how they implement retaliations.  

4. Solve the game of Figure 4.09 in explore-perfect mode and interpret the solutions. Are there any solutions that could be interpreted as generous and contrite?  

5. Modify the game of Figure 4.11 so that probabilistic return from punishment mode is possible. Hint: Make the dfct move from "AC6" go directly to "Some dfct" with a discount factor of 0.9 and payoffs of () for all three players. Then add a chance move from "AD-chance" to "All coop" with some probability and the payoffs () for everyone. Make sure the probabilities from "AD-chance" add up to 0.9 in order to account for discounting. By trial and error, find the maximum probability of return that supports cooperation.