THE RATE LAW IN FLUVIAL GEOMORPHOLOGY

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ABSTRACT. The impact of human activities on geomorphic systems consists primarily of the disruptions of natural processes and the disturbances of steady states. The times required for the systems to achieve new steady states after disruptions (relaxation times) are significant factors in the assessment of man's impact on landform processes. Spatial and dimensional characteristics of fluvial systems can be used as indicators of equilibrium. Dendrochronologic, photographic, and field evidence from gully networks in the Colorado Piedmont indicate that a rate law in the form of a negative exponential function similar to that used for describing relaxation times of radioactive materials and chemical mixtures provides a useful model for relaxation times in geomorphic systems. Half-life values (about 17 yrs in the Denver, Colo. area) for the adjustment periods of the gully networks after natural changes may be useful in predicting adjustment periods after man-induced changes.

The particle and the planet are subject to the same laws, and what is learned of one will be known of the other.

—James Smithson (1765-1829): Engraved on the south face of the Smithsonian Institution, Washington, D.C.

INTRODUCTION

The disruption of geomorphic systems by man's activities is a significant component of human impacts on the natural environment. Fluvial systems are especially susceptible to disruption because of their nearly ubiquitous occurrence and sensitivity to changes in land use. Prior to human interference, stream systems are usually in a condition that approximates a steady state wherein erosion, transport, and deposition are adjusted to the prevailing conditions of climate and geology (Hack, 1960). Human interference in the form of land-use change disturbs the original steady state (Langbein and Leopold, 1964; Howard, 1965). The period between the disruption and the beginning of system change is reaction time (time 'B' in fig. 1); the period between the beginning of change and the establishment of the new steady state is relaxation time (time "C" in fig. 1). Knowledge of the relaxation times for geomorphic systems can be a useful predictive tool in the assessment of environmental impacts of human activities.

In addition, knowledge of relaxation times can be a useful tool for comparing the geomorphic responses of various materials under a variety of climatic conditions. Gullies developed in coarse alluvium in arid regions may erode at different rates than gullies in similar materials under humid conditions, and gullies developed in clay in any environment may behave entirely differently.

Unfortunately, data from geomorphic systems that have responded to human interference are exceptional rather than common, partly because most workers in geomorphology have attempted to avoid man as a variable in their research (notable exceptions include Wolman, 1967;
Fig. 1. Graphic representation of the response of a geomorphic system subjected to disruption. Solid line indicates mean condition, dashed line represents actual values. A system parameter, Y (some dimensional or spatial characteristic of the system such as length or width), is in a steady state during period A. After the climatic or human disruption, new conditions are internalized by the system during period B, the reaction time. During relaxation time, period C, the system adjusts to the new conditions. The rate law provides a model for the system change during this period. A new steady state is established in period D, resulting in new dimensional characteristics as signified by a new mean value for Y.

Not all systems experience steady states. In some cases the initial condition might be in dynamic equilibrium and be drawn as a sloping line in times A and B. After a threshold value is passed, system adjustments in period C would be the same as shown in the diagram. The new conditions in time D might also be represented by a sloping line indicating dynamic equilibrium.

Hammer, 1972; Leopold, 1973; Costa, 1975). Variations in Holocene climate have been numerous, however, and responses of geomorphic systems to climate changes provide clues to potential relaxation times after human disruptions. Geomorphic responses to climatic change are not well understood in many cases, but in some situations the connections may be established. Knox (1972), for example, has suggested that increased sedimentation from agricultural practices provides an analog for the effects of climatic change. If his contention is true, the reverse is also true: climatologically induced changes in fluvial systems are analogs of man induced changes, although the changes may not be of similar magnitude. Wright (1973) makes a similar statement concerning sedimentation in lakes.

Climatic and anthropogenetic changes are remarkably similar in terms of their impacts on geomorphic systems. Both types of changes are relatively rapid in comparison with changes in other environmental systems. The changes of climatic parameters through time are best described by step functions (Bryson and Wendland, 1967), and man has the capability to alter entire drainage basins within a few years. For a dis-
cussion of the rapidity of climatic change, see Baerreis and Bryson (1965) and Wendland and Bryson (1974). Both types of changes affect fluvial systems by changing vegetation and land surfaces, which in turn affect hydrologic regimes. Finally, both climate induced and man induced changes have the potential of affecting complete drainage areas several km² in area which exhibit characteristics of equilibrium.

Woldenberg (1969, 1971, 1972) has analyzed fluvial networks as spatially ordered hierarchies, and he concluded that the networks evolve to form equilibrium hierarchies that reduce frictional loss of energy in the transport of water and detritus. Hooke (1968) has analyzed alluvial fans and their contributing drainage areas, and he concluded that the fans evolve to equilibrium sizes that represent balances between areas of erosion and deposition. Bull (1964, 1975) believes that most alluvial fans approach a state of dynamic equilibrium, where they change in constant proportion to drainage areas. Climatic change and human inference represent two processes that might disrupt these naturally established states of equilibrium, but the rates of adjustment are unknown.

When plotted against time, parameters describing the prevailing climatic and hydrologic conditions respond to interruptions in the form of square wave functions (Knox, 1972), but because of relaxation times, parameters describing vegetation and geomorphic systems respond in the form of square wave lag functions (see fig. 2). Fluvial systems do not immediately shift from one state of equilibrium to another, and the change may vary from one part of the system to another. The adjustment process begins quickly with the geomorphic system approaching the new state of equilibrium at a decreasing rate. For example, Morisawa's investigation of a newly drained lake bed showed that a stream network developed quickly on the newly exposed surface, after which only minor changes occurred (Morisawa, 1964). In most cases, the fluvial systems probably never reach static equilibrium but only closely approximate it in steady states. The term steady state is used in the present context to identify a situation where parameters fluctuate about an unchanging system condition, whereas dynamic equilibrium identifies a situation where parameters fluctuate about a changing system condition (see Chorley and Kennedy, 1972, p. 202-203 for graphic representations and discussion).

As Schumm and Lichty (1965) have pointed out, definition of a steady state condition is dependent on the time scale used. Steady state periods are shorter than geologic or cyclic times, but their precise definition is completely arbitrary. In terms of human perception and interaction with geomorphic systems, if there is no visible change within a geomorphic system in three to four decades, most observers would probably assume equilibrium (Blench, 1957). Also it is possible that some components of geomorphic systems may be in a steady state, while other components are not. In the major example in the present paper, gully length is in a steady
Fig. 2. Parameters \( Y \) of environmental systems responding to disruption: (A) climatic variable such as precipitation, (B) vegetation cover, (C) hillslope potential for fluvial erosion, (D) geomorphic work such as sediment production, (E1) dimensional or spatial characteristics of geomorphic systems that do not have recurring states, (E2) dimensional or spatial characteristics of geomorphic systems that alternate between two states. Curves A and B modified from Bryson and Wendland (1967); C and D from Knox (1972); E1 and E2 by author. Format adopted with changes from Knox (1972).

state, but gully width, depth, or gradient may not be in any state of equilibrium.

By assuming that changes in dimensional or spatial characteristics of geomorphic systems are indicators of disequilibrium, the present paper explores one potential numerical model for describing and evaluating relaxation time. The model discussed below is an attempt to answer the following question: if a fluvial system in a steady state condition is disturbed, how much time is required for the establishment of a new steady state?
A useful model for the description of geomorphic adjustment is the rate law used by physicists and chemists to describe the decay of radioactive isotopes and mixture of solutions. One interpretation of the rate law (a negative exponential decay function) is the half-life concept, wherein exactly half the decaying mass is converted into a new isotope within a specified time period. Within each subsequent equivalent period, half the remaining material decays to the new isotope, and so forth.

The rate law algebraically links the original amount of material with the amount remaining after a given period of decay:

\[(1/2)^{t/T} = A_t/A_o,\]  \hspace{1cm} (1)

where \(t\) = time elapsed, \(T\) = half-life, \(A_t\) = amount of original material remaining at time \(t\), \(A_o\) = amount of original material, and \(1/2\) = rate of change, in this case the amount of decrease in radioactive material in time \(T\) (Feynman, Leighton, and Sands, 1965). Although eq (1) was designed to describe the physical and chemical adjustments of materials and particles, it is also useful for description of the spatial and dimensional adjustments of disrupted geomorphic systems as they adjust to new steady states. In a general way, Smithson (quoted above) foresaw the application in broad contexts of concepts developed for use in describing the behavior of particles.

Other forms of the relationship between changing amounts of material and time are commonly applied by chemists:

\[A_t = A_o e^{-bt},\]  \hspace{1cm} (2)

or:

\[A_x = A_o - A_o e^{-bt},\]  \hspace{1cm} (3)

where \(A_x\) = amount of material changed. Eq (2) can be solved for \(b\), which is the rate constant (Laidler, 1965). The rate constant is easier to interpret if it is resolved into its half-life form by the relationship (Laidler, 1965):

\[T = (\ln 2)/b.\]  \hspace{1cm} (4)

Changes in gully length may be described by the rate law in predicting the relationship between time and gully size. If \(t\) equals time since disruption, \(A_o\) equals the potential equilibrium length of the gully, \(A_x\) equals the length of the gully, and \(A_t\) equals the length yet to be eroded before equilibrium is reached, eq (1) describes the spatial adjustments of a gully system as it erodes to new equilibrium dimensions after some threshold values have been reached for some system component (see Schummm, 1974, for a discussion of thresholds and responses). With the passage of each half-life, half the remaining length of the gully is eroded. Eq (2) describes the length of gully yet to be eroded at a given time, and eq (3) describes gully length at a given time (note that the sum of \(A_x\) and \(A_t\) is \(A_o\)). See Segner (1966) and Megahan (1974) for a different approach to numerical models of gully expansion.
Just as decaying isotopes approach new stable isotopes at continuously decreasing rates, so gullies erode toward equilibrium lengths at continuously decreasing rates. While the lengthening proceeds, other processes such as filling or deepening may take place in the main stem. The equilibrium length is the distance from the mouth of the gully to that point on the thalweg where the amount of flowing water lacks sufficient energy to overcome the resistance of the surface. Gully expansion does not occur upstream from this point. The location of the equilibrium point and definition of the equilibrium length vary according to vegetation cover, geology, soil, and climate. For some applications and at some scales of analysis, the distance from mouth to drainage divide provides a limiting value. The distance from the equilibrium point to the drainage divide, referred to as the critical distance or critical length (Horton, 1945), is especially sensitive to resistance and runoff intensity and has been investigated by other workers (see Horton, 1945, p. 320-323 for a discussion).

Eq (2) may be solved in its linear form:

$$\ln A_t = \ln A_o - bt.$$  \hspace{1cm} (5)

If enough data points are available for $A_t$ (distance from the head of the gully to the equilibrium length or length yet to be eroded) and for $t$ (time since disruption), the solution of the equation defines the rate constant, $b$.

If the exponential form is a workable model for data relating gully length and time and if correlation coefficients for eq (5) are highly significant, the rate law is valid. Interpretation of the function form depends on a thermodynamic and a general systems viewpoint (Strahler, 1952; Chorley, 1962; Smalley and Vita-Finzi, 1969). If the negative exponential form provides the best fit for the data, a negative feedback loop is operating to dampen the effects of disruption, resulting in a decreasing rate of change that approaches some steady state. If the power function or linear function provides the best fit, a dynamic equilibrium situation prevails where a steady state condition is not approached. It should be noted that some authors view the asymptotic approach to a steady state as a special form of dynamic equilibrium. Thermodynamic equilibrium with its associated concept of decay to maximum entropy is best described by a negative exponential form.

**DATA**

Gullies developed in the Colorado Piedmont proved convenient test cases for the evaluation of the rate law and the concept of half-life relaxation times. Gullies in the Piedmont region are in an area that has experienced substantial climatic changes in the past several centuries (Tuan, 1966), and in many areas near Denver, Colo., ponderosa pine trees grow along the floors and sides of the gullies, providing convenient dating mechanisms through dendrochronological analysis. In addition, urban expansion of the city of Denver has introduced human-controlled disrup-
tions to other nearby gully networks, with maps and aerial photography providing long-term observations.

Two gully networks provided data for the analysis of disruption and response (see fig. 3 for locations). They occur in drainage basins approximately 0.85 km² with relief of 45 m. Gullies in the first network are located in a rural setting 27 km southeast of downtown Denver, in rolling terrain in sec. 24, NE¼SE¼, T.5S. R.66W., shown on the Piney Creek, Colo., Quadrangle (U.S. Geol. Survey, scale 1:24,000). The gully network has two major trunks about 0.8 km long with numerous smaller tributary gullies, all apparently eroded in response to climatic changes (Hunt, 1954; Scott, 1963), because their initiation dates do not corres-

Fig. 3. Locations of the urban and rural gullied basins.
pond with known dates of disruption. The second gully network consisting of a single trunk stream 1.3 km long with several tributaries is located in an urban area 16 km north of downtown Denver in the suburbs of Northglenn, Thornton, and Federal Heights. It is located in T.2S. R.68W. at the junction of sec. 20, 21, 28, and 29 as shown on the Arvada, Commerce City, East Lake, and Lafayette, Colo., Quadrangles (U.S. Geol. Survey, scale 1:24,000). Both networks are eroded into the Denver and Arapaho formations, sandy alluvial deposits of Paleocene age (Thornbury, 1965). Construction activities have caused accelerated erosion in the area of the suburban networks.

In the rural network, two gully systems have developed, one inside the other. The gully sides have clearly defined, matched shoulders so that in cross section the gullies represent small trenches cut into larger trenches etched into the smoothly sloping landscape (fig. 4). There is no perceptible lithologic or structural control to explain the bench-like morphology that is clearly of erosional origin since the shoulders are cut into the Paleocene alluvium that forms the area’s bedrock. The total gully morphology suggests episodic erosion, with the initial creation of a protogully that was later dissected by a second gully. Dendrochronological evidence confirms

![Fig. 4. A typical cross section of the rural gully and protogully. The modern alluvial deposits in the channel are partly buried by slump and soil-fall deposits that will be washed away during the next flood. The ponderosa pine on the left is the older of the two trees and is growing in the floor of the protogully. The dashed line indicates the apparent morphology of the protogully before incision by a newer gully.](image-url)
that the benches are substantially older than the present gully floors, which are blanketed by alluvial material similar to the late post-Piney Creek Alluvium described by Scott (1963) and by Maberry and Lindvall (1972) for nearby areas.

The meager sedimentary deposits of the study area do not clearly record the activity of the fluvial systems in the late Holocene, but the erosional record is preserved by the valley/gully forms and the trees growing in them. As the protogully developed, newly exposed surfaces were colonized by ponderosa pine, so that now the oldest trees grow in the lower reaches of the system, since these surfaces were formed first. Younger trees grow in the headward parts of the system, because these areas were exposed last. The trees also grow in the bottom of the gully and show a similar progression from oldest to youngest with increasing distance from the mouth of the system.

The trees provide a convenient dating mechanism with which to analyze the development of the protogully and the gully, because the age of each tree provides a minimum date for the establishment of the surface on which it grows, and trees on the valley bottom provide indicators for the minimum ages of the bottom surface at numerous points along the profile. Thus, the minimum ages and rates of development for the protogully and gully systems can be established and compared with man's activities in the area or with climatic trends as recorded in tree-ring data from upland sites between the gullies.

Although 78 samples were taken with a Swedish increment borer, only the oldest tree in each 15 m reach of each gully was used. Gully length was measured as the distance from the mouth of the gully to the sampled tree or to the center of the top of the headcut.

RESULTS

The tree ring evidence indicates that the protogully system began its development in 1826 and that by 1880 the headcut had eroded to 85 percent of its ultimate length. Trees growing at and near the present head of the protogully system show that it has been stable since about 1927. The distributions of ages along the profiles indicate rapid initial growth of the system, followed by a much slower asymptotic approach to an apparent equilibrium state. A ponderosa pine over 40 yrs old is growing 2 m downvalley from the headcut as an indicator of stability.

Trees growing in the bottoms show that the rural gully system began erosional development in or slightly earlier than 1906. The gully had a developmental history very similar to its immediate predecessor, with rapid initial growth, continuing reduction in the rate of development, and finally an asymptotic approach to what appears to be an equilibrium state where the gully length is in balance with the hydrologic forces that shape it. Vegetation immediately downvalley from the headcut indicates stability.

The primary factor in the initiation of gully erosion in the study area appears to be increased summer rainfall; at least the starting dates
of gully development coincide with the starting dates of extended periods of more intense rainfall as reflected in the dendrochronology of many areas of Colorado, New Mexico, and Arizona (Schulman, 1942, 1945; Fritts, 1965). Growth rings of trees in the study area support Schumm and Hadley's (1957) conclusion that since the fourteenth century, major periods of intense summer rainfall began in 1825 and 1905, dates that are remarkably close to the minimum starting dates of 1826 for the protogully and 1906 for the gully. It is unlikely that man's mismanagement of land surface was responsible for accelerated erosion in these cases because European settlement of the study area had not yet begun in 1826, and there were no notable changes in the numbers of cattle grazing the ranges in the first decade of the twentieth century (Peterson, 1950).

Schumm (1974) and Weaver and Schumm (1975) have suggested that the discontinuous downcutting may simply be inherent in the gullying process, and if this is the case, the rural gullies may not be related to climatic change. Data in the present limited study does not resolve the question, but the initiation dates strongly suggest that climatic change was at least a trigger mechanism.

Aerial photography suggests that the urban gully began in 1960, about the time that suburban development began in the area. Construction of streets, highways, residences, and a small reservoir have contributed to the surficial disruption of the basin. Field observations indicate that the gully is still actively expanding. The steady state point, which has yet to be reached, is assumed to be in a topographic location similar to the position in the rural gullies because the geologic, geomorphic, vegetative, and climatic conditions are similar.

Using the values for $A$, and $t$ in eq (5) from dendrochronology, aerial photography, and field evidence, the constants $b$ and $A$, in eq (5) were evaluated using the least squares technique (table 1, fig. 5). Very high correlation coefficients for all three gully systems suggest that the general exponential model is a useful representation of the relaxation times of gully systems.

A power function model has frequently been used by previous workers to describe dimensional relationships in systems with equilibrium conditions (for example, Leopold and Maddock, 1953; Hooke, 1968). The power function model may be interpreted as an expression of allometric change (as by Bull, 1975), but in a classical sense this interpretation holds only when physical dimensional variables are related to each other. The concept of allometry does not relate rates of change to time, so that the rate law may be considered as a separate set of relationships. A most effective description of system change would employ a combination of the concept of allometric change and the concept of the rate law.

The negative exponential form for the rate law and its close agreement with the data indicate that spatial or dimensional adjustments in the gully systems take place at a decreasing rate, suggesting the operation of a negative feedback loop. As the gully length increases, the head-
Table 1

<table>
<thead>
<tr>
<th>Network</th>
<th>Area</th>
<th>Disruption Type</th>
<th>Date</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Protogully</td>
<td>Rural</td>
<td>Natural</td>
<td>1826</td>
<td>16</td>
</tr>
<tr>
<td>Gully</td>
<td>Rural</td>
<td>Natural</td>
<td>1906</td>
<td>20</td>
</tr>
<tr>
<td>Gully</td>
<td>Urban</td>
<td>Human</td>
<td>1960</td>
<td>10</td>
</tr>
</tbody>
</table>

cut retreats farther into the headwater areas of the system where lesser discharges and lower amounts of energy are available for erosion, so that increasing length dictates decreasing erosion rates. The result of this self-regulating negative feedback loop is that migration rates for the headcuts are eventually reduced nearly to zero, and the new equilibrium state is approached in extremely small increments.

Using equations (2), (3), and (5), it is possible to calculate the value of $T$, the half-life, of each system using the same data base. The approximately steady state length of the rural gullies can be identified within a few meters in the field because of the growth of trees several decades old within a few meters of the headcut. The half-lives calculated using eq (4) and given in table 1 indicate that if a small fluvial system in the Denver area is disrupted by climatic change or human activities, the

Fig. 5. Dendrochronological data from trees growing on the floors of the rural gully and protogully.
system will adjust by eroding a gully toward a new steady state length and by approaching half that length in about 17 yrs. Three-quarters of the length to a steady-state dimension will be eroded in 34 yrs, seven-eights in 51 yrs, and so on. Comparison of these half-life values with values from gullies developed in other climatic and geologic environments can provide a useful perspective on gullying processes. The rate law alone can not be used to analyze the ultimate sizes or shapes of new erosion forms, issues most likely to be resolved by application of concepts of allometric change.

CONCLUSIONS

Although the results from the three test gullies conform closely to the rate law and the half-life concept of system adjustment, the universality of the principles remains to be proven. The three systems reviewed in table 1 probably have half-lives that are almost the same because they developed under similar geologic, vegetative, and climatic conditions. Other networks developed under different conditions probably have different relaxation times and thus different half-lives. Additionally, reaction times for the gullies remain unknown, though in the urban gullies evidence from aerial photos suggests that it is on the order of 0.5 to 1.5 yrs.

The rate law and the half-life concepts indicate that following disruptions, geomorphic systems approach new steady states very rapidly at first, but that adjustment becomes progressively slower. The form of adjustment may be explained by the distribution of available energy in the stream system, with smallest amounts in the headwater regions. Viewed from this perspective, human disruptions of geomorphic systems in the form of altered surface conditions are just one of many types of interruption, with the systems responding in a predictable manner. If the half-life of a geomorphic system in given geologic and climatic conditions is known, predictions of the environmental impact of human activities can be quantified. In many cases, the half-lives must be determined from evidence of non-human disruptions, and in most cases the disruptions are manifested as changes in the spatial or dimensional characteristics of the systems.

The evidence presented above indicates that the answer to the question — how much time is required for geomorphic adjustment after disruption? — is not the same for all systems, but the same model — the rate law — provides a general framework for prediction and comparison.
ACKNOWLEDGMENTS

Darrel P. Eyman, Department of Chemistry, University of Iowa, commented on an early draft of the present paper and supplied many useful insights into the use of the rate law by chemists and physicists. Discussions with Michael Woldenberg, Department of Geography, State University of New York were very useful, and Stuart A. Klugman, Department of Statistics, University of Iowa, also provided helpful suggestions. Preparation of the manuscript benefited substantially from the thoughtful criticism of William B. Bull of the University of Arizona, Roger Hooke of the University of Minnesota, R. J. Parker of the U.S. Geological Survey, Marie Morisawa of the State University of New York at Binghamton, Roger N. Dubois of the University of Maryland, Baltimore County, and Wayne N. Engstrom, California State University, Fullerton. W. C. Stanley drafted figure 3.

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The rate law in fluvial geomorphology


